Chapter 2

Fluid Flow in Food Processing

In any commercial food processing plant, the movement of liquid foods from one location to another becomes an essential operation. Various types of systems are used for moving raw or unprocessed liquid foods as well as processed liquid products before packaging. The range of liquid foods encountered in a processing plant is extremely wide, encompassing foods with distinctly different flow properties, from milk to tomato paste. The design of these systems in food processing is significantly different from most other applications because of the essential need for sanitation to maintain product quality. The transport system must be designed to allow for ease and efficiency in cleaning.

In this chapter, we will concern ourselves mostly with the flow of fluids. Fluid is a general term used for either gases or liquids. Most of our discussion will deal with liquid foods. A fluid begins to move when a force acts upon it. At any location and time within a liquid transport system, several types of forces may be acting on a fluid, such as pressure, gravity, friction, thermal effects, electrical charges, magnetic fields, and Coriolis forces. Both the magnitude and direction of the force acting on a fluid are important. Therefore, a force balance on a fluid element is essential to determine which forces contribute to or oppose the flow.

From our daily experience with handling different kinds of fluids, we know that if pressure at one location within a fluid system is higher than another, the fluid moves toward the region of lower pressure. Gravity causes the flow of fluid from higher to lower elevations. A fluid moving to a lower elevation undergoes a decrease in its potential energy, while its kinetic energy increases. With the presence

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Conceptually we may visualize that inside a fluid in motion one imaginary layer of fluid is sliding over another. The viscous forces act tangentially on the area between these imaginary layers, and they tend to oppose the flow. This is why if you spill honey—a highly viscous food—it moves much more slowly than milk, which has a substantially lower viscosity. All fluids exhibit some type of viscous behavior distinguished by a flow property called viscosity. We will examine these factors and their role in the design of equipment for transporting different types of liquid foods and ingredients to different locations within a processing plant.

2.1 LIQUID TRANSPORT SYSTEMS

A typical transport system consists of four basic components, namely, tanks, pipeline, pump, and fittings. Figure 2.1 illustrates a simple milk pasteurization line. Raw milk enters the balance tank prior to the pasteurization process and finally exits from the flow diversion valve. Between the tank and the valve is the conduit, or pipeline, for milk flow. Unless flow can be achieved by gravity, the third primary component is the pump, where mechanical energy is used for product transport. The fourth component of the system consists of fittings such as valves and elbows, used to control and direct flow. The tanks used in these types of systems may be of any size and configuration. In addition to the basic components of a transport system, there may be additional processing equipment as part of the system, such as a heat exchanger to pasteurize milk, as shown in Figure 2.1.

2.1.1 Pipes for Processing Plants

Fluids (liquids and gases) in food processing plants are transported mostly in closed conduits—commonly called pipes if they are round, or ducts if they are not round. Although sometimes used in processing plants, open drains generally are avoided, for sanitation reasons. The pipelines used for liquid foods and their components have numerous unique features. Probably the most evident feature is the use of stainless steel for construction. This metal provides smoothness, cleanability, and corrosion resistance. The corrosion resistance of stainless steel is attributed to "passivity"—the formation of a surface film on the metal surface when exposed to air. In practice, this surface film must reform

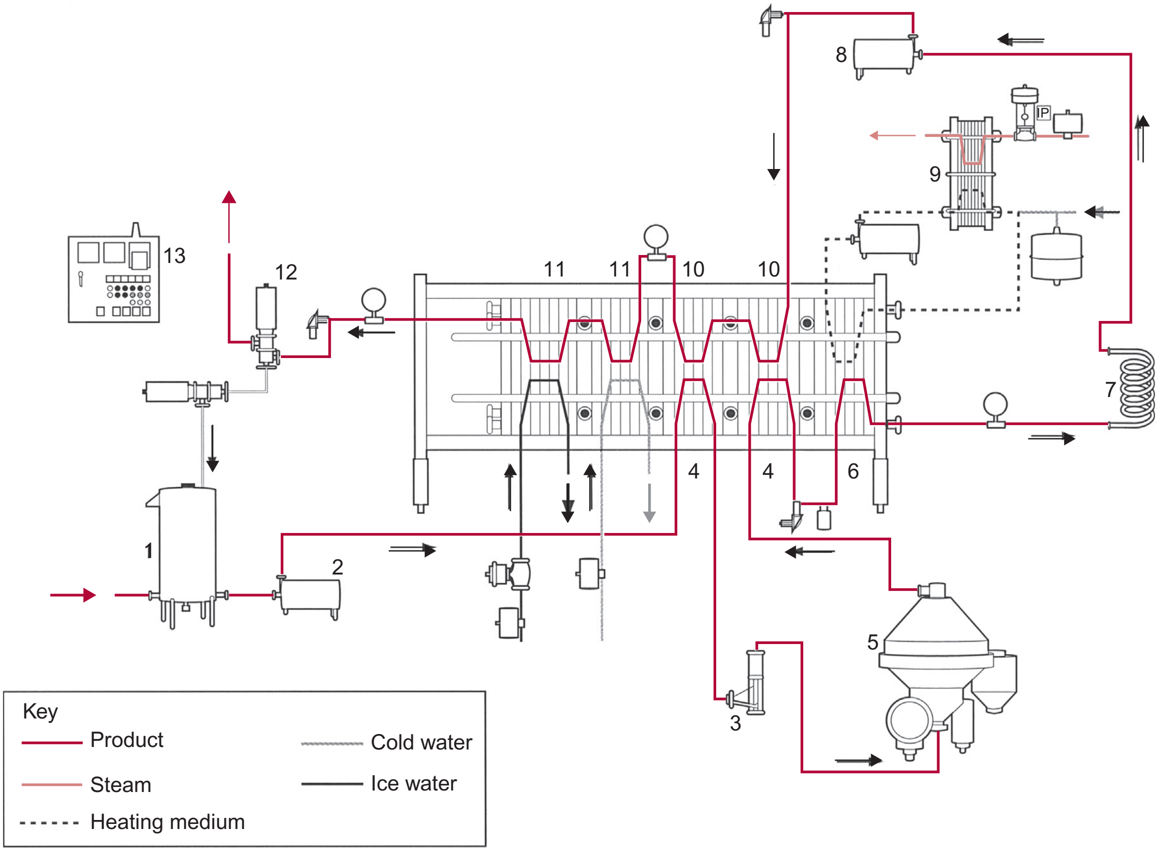
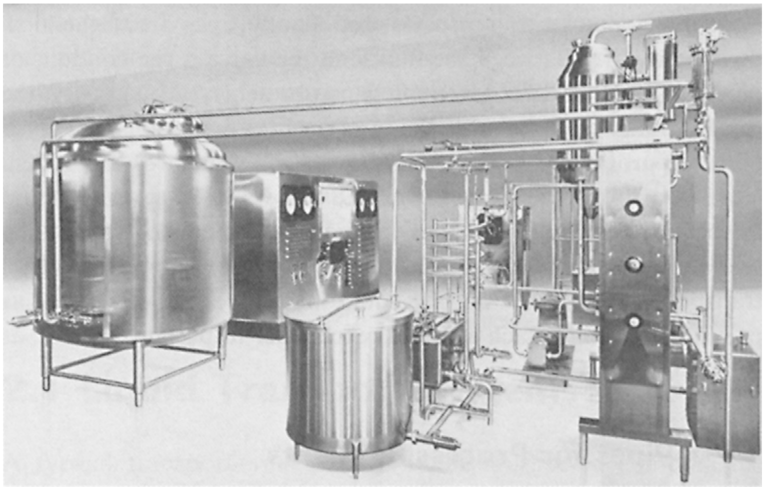


Figure 2.1 Production line for milk processing. (1) Balance tank, (2) feed pump, (3) flow controller, (4) regenerative preheating section, (5) centrifugal clarifier, (6) heating section, (7) holding tube, (8) booster pump, (9) hot water heating system, (10) regenerative cooling sections, (11) cooling sections, (12) flow diversion valve, (13) control panel. (Courtesy of Tetra Pak Processing Systems AB)

each time after the surface is cleaned. If the protective surface film is impaired, which could occur from failure to establish passivity or any action resulting in film removal, the site is susceptible to corrosion. Therefore, stainless-steel surfaces require care to maintain corrosion resistance, especially after cleaning. A detailed description of corrosion mechanisms is given by Heldman and Seiberling (1976).

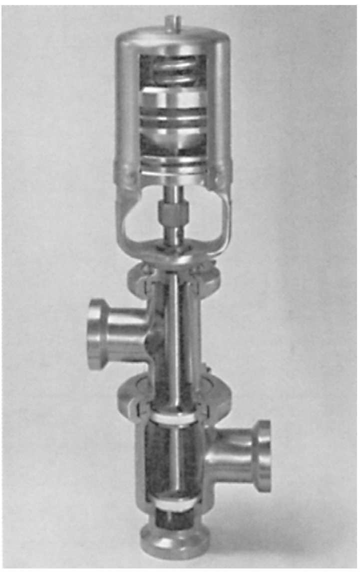
A typical pipeline system for liquid food transport contains several essential components. In addition to the straight lengths of the pipe, which may vary in diameter from 2 to 10 cm, elbows and tees are essential for changing the direction of product movement. As shown



■ Figure 2.2 A typical liquid food processing system, illustrating pipelines and pipeline components. (Courtesy of CREPACO, Inc.)

in Figure 2.2, these components are welded into the pipeline system and may be used in several different configurations. Another component is the valve used to control the flow rate; an air-actuated valve is illustrated in Figure 2.3. This valve may be operated remotely, often based on some type of preset signal.

It is essential that all components of the pipeline system contribute to sanitary handling of the product. The stainless-steel surfaces ensure smoothness needed for cleaning and sanitizing. In addition, proper use of the system provides the desired corrosion prevention. Since cleaning of these systems most often is accomplished by cleaning-in-place (CIP), we must account for this factor in the initial design of the system.



■ Figure 2.3 An air-actuated valve for liquid foods. (Courtesy of Cherry-Burrell Corporation)

2.1.2 Types of Pumps

Except for situations where gravity can be used to move liquid products, some type of mechanical energy must be introduced to overcome the forces opposing transport of the liquid. The mechanical energy is provided by the pumps. There are numerous types of pumps used in the industry. As shown in Figure 2.4, pumps may be classified as **centrifugal** or **positive displacement**. There are variations within each of these types, as shown in the figure.

2.1.2.1 *Centrifugal Pumps*

The use of centrifugal force to increase liquid pressure is the basic concept associated with operation of a centrifugal pump. As illustrated

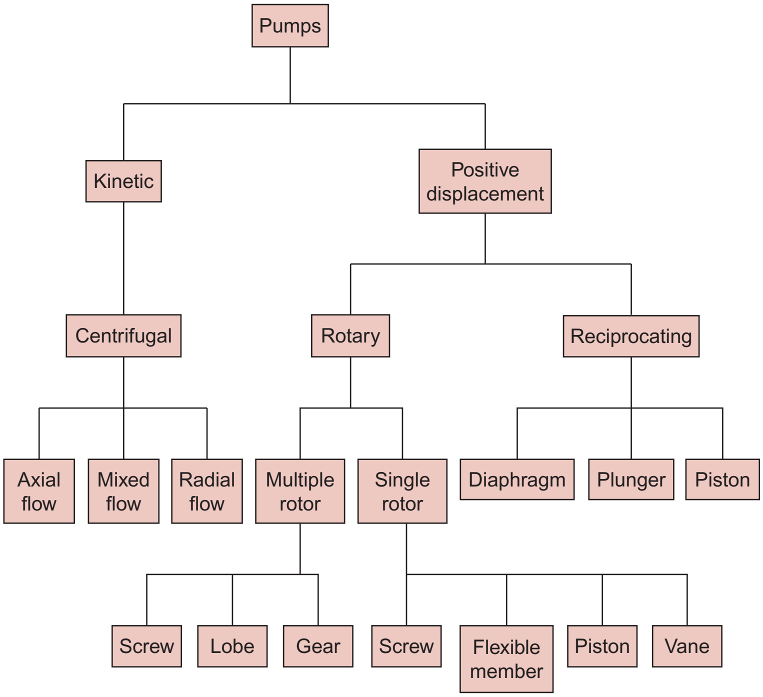


Figure 2.4 Classification of pumps.

in Figure 2.5, the pump consists of a motor-driven impeller enclosed in a case. The product enters the pump at the center of impeller rotation and, due to centrifugal force, moves to the impeller periphery. At this point, the liquid experiences maximum pressure and moves through the exit to the pipeline.

Most sanitary centrifugal pumps used in the food industry use two-vane impellers (Fig. 2.5). Impellers with three and four vanes are available and may be used in some applications. Centrifugal pumps are most efficient with low-viscosity liquids such as milk and fruit juices, where flow rates are high and pressure requirements are moderate. The discharge flow from centrifugal pumps is steady. These pumps are suitable for either clean and clear or dirty and abrasive liquids. They are also used for pumping liquids containing solid particles (such as peas in water). Liquids with high viscosities, such as honey, are difficult to transport with centrifugal pumps, because they are prevented from attaining the required velocities by high viscous forces within the product.

Flow rates through a centrifugal pump are controlled by a valve installed in the pipe and connected to the discharge end of the pump.

This approach provides an inexpensive means to regulate flow rate, including complete closure of the discharge valve to stop flow. Since this step will not damage the pump, it is used frequently in liquid food processing operations. However, blocking flow from a centrifugal pump for long periods of time is not recommended, because of the possibility of damage to the pump. The simple design of the centrifugal pump makes it easily adaptable to cleaning-in-place functions.

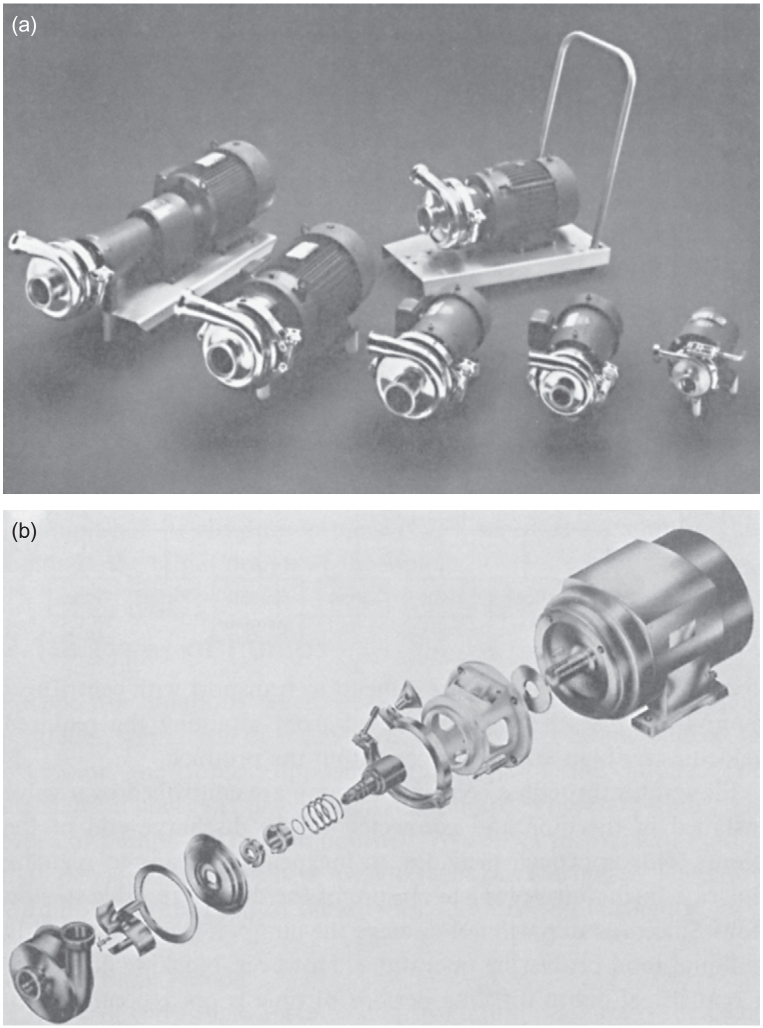


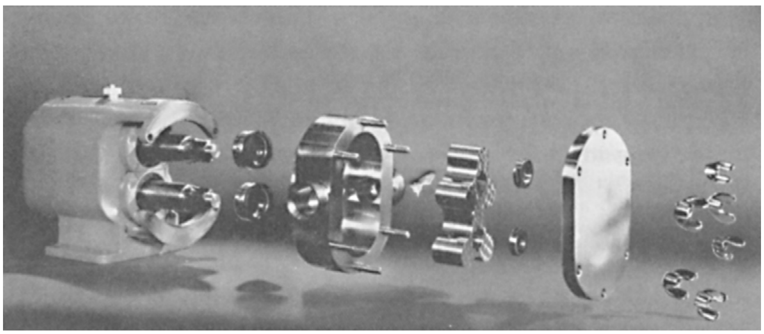
Figure 2.5 (a) Exterior view of a centrifugal pump. (Courtesy of Cherry-Burrell Corporation) (b) A centrifugal pump with components. (Courtesy of CREPACO, Inc.)

2.1.2.2 Positive Displacement Pumps

By application of direct force to a confined liquid, a positive displacement pump produces the pressure required to move the liquid product. Product movement is related directly to the speed of the moving parts within the pump. Thus, flow rates are accurately controlled by the drive speed to the pump. The mechanism of operation also allows a positive displacement pump to transport liquids with high viscosities.

A **rotary pump**, illustrated in Figure 2.6, is one type of positive displacement pump. Although there are several types of rotary pumps, the general operating concept involves enclosure of a pocket of liquid between the rotating portion of the pump and the pump housing. The pump delivers a set volume of liquid from the inlet to the pump outlet. Rotary pumps include sliding vane, lobe type, internal gear, and gear type pumps. In most cases, at least one moving part of the rotary pump must be made of a material that will withstand rubbing action occurring within the pump. This is an important feature of the pump design that ensures tight seals. The rotary pump has the capability to reverse flow direction by reversing the direction of rotor rotation. Rotary pumps deliver a steady discharge flow.

The second type of positive displacement pump is the **reciprocating pump**. As suggested by the name, pumping action is achieved by application of force by a piston to a liquid within a cylinder. The liquid moves out of the cylinder through an outlet valve during forward piston movement. Reciprocating pumps usually consist of several cylinder–piston arrangements operating at different cycle positions to ensure more uniform outlet pressures. Most applications are for low-viscosity liquids requiring low flow rates and high pressures. The reciprocating pumps deliver a pulsating discharge flow.



■ **Figure 2.6** A positive displacement pump with illustration of internal components. (Courtesy of Tri-Canada, Inc.)

2.2 PROPERTIES OF LIQUIDS

The transport of a liquid food by one of the systems described in the previous section is directly related to liquid properties, primarily viscosity and density. These properties will influence the power requirements for liquid transport as well as the flow characteristics within the pipeline. An understanding of the physical meaning associated with these properties is necessary in order to design an optimal transport system. Later in this chapter, we will examine different approaches used for measurement of these properties.

2.2.1 Terminology Used in Material Response to Stress

Fluid flow takes place when force is applied on a fluid. Force per unit area is defined as **stress**. When force acting on a surface is perpendicular to it, the stress is called **normal stress**. More commonly, normal stress is referred to as **pressure**. When the force acts parallel to the surface, the stress is called shear stress, σ. When shear stress is applied to a fluid, the fluid cannot support the shear stress; instead the fluid deforms, or simply stated, it flows.

The influence of shear stress on solids and liquids leads to a broad classification of such materials as plastic, elastic, and fluid.

In the case of an **elastic** solid, when shear stress is applied, there is a proportional finite deformation, and there is no flow of the material. On removal of the applied stress, the solid returns to its original shape.

A **plastic** material, on the other hand, deforms continuously on application of shear stress; the rate of deformation is proportional to the shear stress. When the shear stress is removed, the object shows some recovery. Examples include Jell-O® and some types of soft cheese.

A **fluid** deforms continuously on application of shear stress. The rate of deformation is proportional to the applied shear stress. There is no recovery; that is, the fluid does not retain or attempt to retain its original shape when the stress is withdrawn.

When normal stress or pressure is applied on a liquid, there is no observed appreciable effect. Thus, liquids are called **incompressible** fluids, whereas gases are **compressible** fluids, since increased pressure results in considerable reduction in volume occupied by a gas.

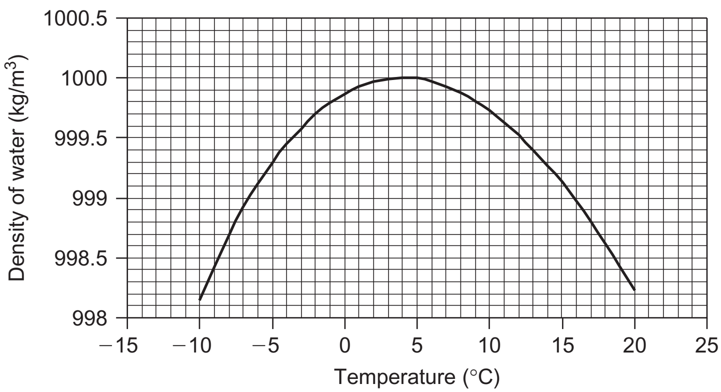
2.2.2 Density

The density of a liquid is defined as its mass per unit volume and is expressed as kg/m^3 in the SI unit system. In a physical sense, the magnitude of the density is the mass of a quantity of a given liquid occupying a defined unit volume. The most evident factor is that the magnitude of density is influenced by temperature. For example, the density of water is maximum at 4°C and decreases consistently with increasing temperature (Fig. 2.7).

Densities of liquids are most often measured by a hand hydrometer. This instrument measures specific gravity, which is the ratio of the density of the given liquid to the density of water at the same temperature. The measuring instrument is a weighted float attached to a small-diameter stem containing a scale of specific-gravity values. The float will sink into the unknown liquid by an amount proportional to the specific gravity, and the resulting liquid level is read on the stem scale. However, when converting specific gravity values to density, care must be taken to ensure that the value for density of water at the measurement temperature is used.

2.2.3 Viscosity

A fluid may be visualized as matter composed of different layers. The fluid begins to move as soon as a force acts on it. The relative movement of one layer of fluid over another is due to the force, commonly called shearing force, which is applied in a direction parallel to the surface over which it acts. From Newton's second law of motion, a resistance force is offered by the fluid to movement, in the opposite direction to the shearing force, and it must also act in a



■ **Figure 2.7** Density of water as a function of temperature.

direction parallel to the surface between the layers. This resistance force is a measure of an important property of fluids called **viscosity**.

With different types of fluids, we commonly observe a wide range of resistance to movement. For example, honey is much more difficult to pour out of a jar or to stir than water or milk. Honey is considerably more viscous than milk. With this conceptual framework, we will consider a hypothetical experiment.

Consider two parallel plates that are infinitely long and wide separated by a distance dy, as shown in Figure 2.8a. First, we place a solid block of steel between the two plates and firmly attach the steel block to the plates so that if we move a plate, the attached surface of steel block must also move with it. The bottom plate is then anchored so that it remains fixed during the entire experiment. Next, we apply a force, F, to the top plate so that it moves by a small distance, δx, to the right. Due to this displacement of the plate, an imaginary line in the steel block, AC, will rotate to AC′, and the angle of deflection will be $\delta\theta$. The force in the steel resisting the movement will be acting at the steel–plate interface, in the direction opposite to the applied force F. This opposing force acts on area A, the contact area between the plate and the steel block. The opposing force equals σA, where σ is the shear stress (force per unit area). Experimental evidence indicates that for solid materials such as steel, the angular deflection $\delta\theta$ is proportional to the shearing stress σ. When the force is removed, the steel block returns to its original shape. Therefore, steel is called an elastic material.

If we carry out the same experiment with a fluid inserted between the two plates instead of steel (Fig. 2.8b), our observations will be remarkably different. The bottom plate is anchored and remains fixed throughout the experiment. We apply force F to the top plate. After a short, transient interval, the top plate will continue to move with a velocity du as long as the force keeps acting on the top plate. The fluid layer immediately below the top plate actually "sticks" to it and moves to the right with a velocity du, whereas the bottom-most layer sticks to the bottom fixed plate and remains stationary. Between these two extreme layers, the remaining layers will also move to the right, with each successive layer dragged along by the layer immediately above it. A velocity profile, as shown in Figure 2.8b, develops between the top and bottom plate. This situation is analogous to the deck of cards shown in Figure 2.9. If the top card is moved to the right, it drags the card immediately below it, and that card

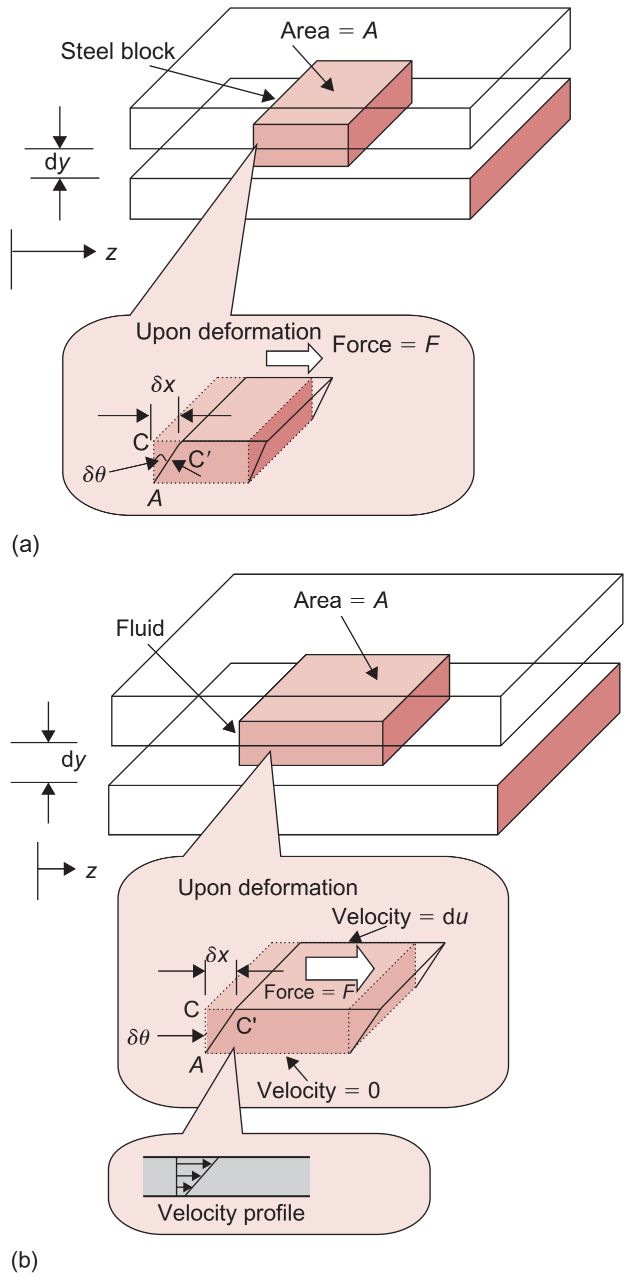
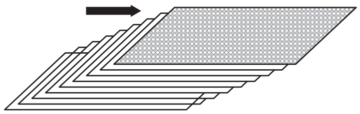


Figure 2.8 (a) A steel block enclosed between two plates. (b) A fluid enclosed between two plates.



■ Figure 2.9 Illustration of drag generated on underlying cards as the top card in a deck is moved. This is analogous to the movement of the top layer of a fluid.

drags the one below it, and so on. The drag force depends upon the frictional resistance offered by the surface in contact between the cards.

Referring to Figure 2.8b, if, in a small increment of time δt, the line AC′ deflects from AC by an angle $\delta\theta$, then

$$\tan \delta\theta = \frac{\delta x}{\mathrm{d}y} \tag{2.1}$$

For a small angular deflection,

$$\tan \delta\theta \approx \delta\theta \tag{2.2}$$

Therefore,

$$\delta\theta = \frac{\delta x}{\mathrm{d}y} \tag{2.3}$$

But, linear displacement, δx, is equal to the product of velocity and time increment, or

$$\delta x = \mathrm{d}u\ \delta t \tag{2.4}$$

Therefore,

$$\delta\theta = \frac{\mathrm{d}u\ \delta t}{\mathrm{d}y} \tag{2.5}$$

Equation (2.5) implies that the angular displacement depends not only on the velocity and separation between the plates, but also on time. Therefore, in the case of fluids, the shearing stress must be correlated to the rate of shear rather than shear alone, as was done for solid materials. The rate of shear, $\dot{\gamma}$, is

$$\dot{\gamma} = \lim \frac{\delta\theta}{\delta t} \tag{2.6}$$

or

$$\dot{\gamma} = \frac{\mathrm{d}u}{\mathrm{d}y} \tag{2.7}$$

Thus, shear rate is the relative change in velocity divided by the distance between the plates. Newton observed that if the shearing stress σ is increased (by increasing force, F), then the rate of shear, $\dot{\gamma}$, will also increase in direct proportion.

$$\sigma \propto \dot{\gamma} \tag{2.8}$$

or

$$\sigma \propto \frac{\mathrm{d}u}{\mathrm{d}y} \tag{2.9}$$

Or, removing the proportionality by introducing a constant, μ,

$$\sigma = \mu \frac{\mathrm{d}u}{\mathrm{d}y} \tag{2.10}$$

where μ is the coefficient of viscosity, or simply viscosity, of the fluid. It is also called "absolute" or "dynamic" viscosity.

Liquids that follow Equation (2.10), exhibiting a direct proportionality between shear rate and shear stress, are called Newtonian liquids. When shear stress is plotted against shear rate, a straight line is obtained passing through the origin (Fig. 2.10). The slope of the line gives the value for viscosity, μ. Water is a Newtonian liquid; other foods that exhibit Newtonian characteristics include honey, fluid milk, and fruit juices. Table 2.1 gives some examples of coefficients of viscosity. Viscosity is a physical property of the fluid and it describes the resistance of the material to shear-induced flow. Furthermore, it depends on the physicochemical nature of the material and the temperature.

Liquids that do not follow Equation (2.10) are called non-Newtonian liquids, and their properties will be discussed later in Section 2.9.

Table 2.1 The Viscosity of Some Common Materials at Room Temperature

Liquid	Viscosity, approximate(Pa s)
Air	10^{-5}
Water	10^{-3}
Olive oil	10^{-1}
Glycerol	10^{0}
Liquid honey	10^{1}
Golden syrup	10^{2}
Glass	10^{40}

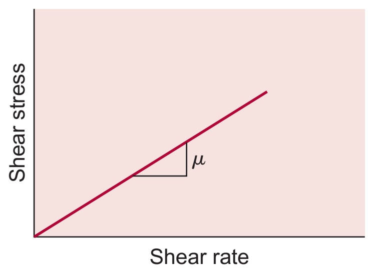


Figure 2.10 Shear stress vs shear rate behavior for a Newtonian fluid.

Shear stress σ is obtained using Equation (2.10). Since force is expressed by N (newtons) and area by m^2 (meters squared), shear stress is expressed in units of Pa (pascal) as follows,

$$\sigma \equiv \frac{\mathrm{N}}{\mathrm{m}^2} \equiv \mathrm{Pa}$$

Note that it is common to see τ used as a symbol for shear stress in literature. However, because the Society of Rheology recommends the use of σ for shear stress, we will use this symbol consistently in this book. In the cgs units, shear stress is expressed as dyne/cm^2, where,

$$1\ \mathrm{Pa} \equiv 10\ \mathrm{dyne/cm^2}$$

The term $\dot{\gamma}$, or du/dy in Equation (2.10), is called rate of shear or shear rate. It is the velocity gradient set up in the liquid due to the applied shear stress as seen in Equation (2.7). Its units are s^{-1}, determined by dividing the change in velocity (m/s) by distance (m). Therefore, the unit of viscosity, μ, in the SI system of units is pascal-second (Pa s) obtained as follows.

$$\mu \equiv \frac{\sigma}{\dot{\gamma}} \equiv \frac{\mathrm{Pa}}{\mathrm{s}^{-1}} \equiv \mathrm{Pa\ s}$$

Frequently, the viscosity of liquids is expressed as millipascal-second or mPa s, where

$$1000\ \mathrm{mPa\ s} = 1\ \mathrm{Pa\ s}$$

The unit Pa s may also be expressed as:

$$\mu \equiv \mathrm{Pa\ s} \equiv \left(\frac{\mathrm{N}}{\mathrm{m}^2}\right)\mathrm{s} \equiv \left(\frac{\mathrm{kg\ m}}{\mathrm{s}^2\mathrm{m}^2}\right) \equiv \frac{\mathrm{kg}}{\mathrm{m\ s}}$$

In cgs units, where shear stress in dyne/cm^2 and shear rate is s^{-1}, viscosity is expressed as Poise (named after Poiseulle), or

$$\mu \equiv \frac{\mathrm{dyne\ s}}{\mathrm{cm}^2} \equiv \mathrm{poise}$$

In literature, viscosity of liquids is often expressed in centipoise (0.01 poise). Note the following conversion factors:

$$1\ \mathrm{poise} \equiv 0.1\ \mathrm{Pa\ s}$$

and

$$1\ \mathrm{cP} = 1\ \mathrm{mPa\ s}$$

The viscosity of water at ambient temperatures is about 1 centipoise (or 1 mPa s), whereas the viscosity of honey is 8880 centipoise.

According to Van Wazer (1963), the human eye can distinguish between differences in viscosity of fluids in the range of 100 to 10,000 cP. Above 10,000 cP the material appears to be like a solid. Thus, a liquid with a viscosity of 600 cP will appear twice as "thick" as a liquid with a viscosity of 300 cP.

Although dynamic viscosity, μ, is commonly used, an alternative term used to express viscosity is kinematic viscosity, ν. When Newtonian liquids are tested by capillary viscometers such as Ubbeholde or Cannon-Fenske (to be discussed later in Section 2.8.1), the force of gravity is used to move a liquid sample through a capillary. Therefore, the density of the liquid plays an important role in calculations. Kinematic viscosity is commonly used to express viscosity of nonfood materials such as lubricating oils. It is related to dynamic viscosity as follows:

$$\text{Kinematic viscosity} = \frac{\text{dynamic viscosity}}{\text{density}}$$

or

$$\nu = \frac{\mu}{\rho} \tag{2.11}$$

The units of kinematic viscosity are

$$\nu \equiv \frac{\text{m}^2}{\text{s}}$$

In cgs units, kinematic viscosity is expressed in units of stokes (abbreviated as S) or centistokes (cS). This unit is named after Sir George Stokes (1819–1903), a Cambridge physicist who made major contributions to the theory of viscous fluids. Where

$$1\text{S} = 100\ \text{cS}$$

or

$$1\ \text{cS} = \frac{1\ \text{mm}^2}{\text{s}}$$

water has a kinematic viscosity of 1 mm^2/s at 20.2°C.

Example 2.1

In a quality control test, viscosity of a liquid food is being measured with a viscometer. A shear stress of 4 dyne/cm^2 at a shear rate of 100 s^{-1} was recorded. Calculate the viscosity and express it as Pa s, cP, P, kg/m s and mPa s.

Given

Shear stress $= 4$ *dyne/cm*^{2}
Shear rate $= 100\ s^{-1}$

Approach

We will use the definition of viscosity given in Equation (2.10) to calculate the viscosity. For unit conversions, note that

$$1\ dyne/cm^2 = 1\ g/(cm\ s^2) = 0.1\ kg/(m\ s^2) = 0.1\ N/m^2 = 0.1\ Pa$$

Solution

1. *Shear stress in SI units is*

$$\sigma = \frac{4\,[dyne/cm^2] \times 0.1\,[kg/(m\ s^2)]}{1\,[dyne/cm^2]}$$

$$\sigma = 0.4\ kg(m\ s^2)$$

$$\sigma = 0.4\ Pa$$

2. *Viscosity in Pa s*

$$\mu = \frac{0.4\,[Pa]}{100\,[s^{-1}]} = 0.004\ Pa\ s$$

3. *Viscosity in P*

$$\mu = \frac{4\,[dyne/cm^2]}{100\,[s^{-1}]}$$

$$= \frac{0.04\,[dyne\ s)/cm^2]}{1\,[dyne\ s/cm^2]/1\,[P]} = 0.04\ P$$

4. *Viscosity in cP*

$$\mu = \frac{0.04\,[P]}{1\,[P]/100\,[cP]}$$

$$\mu = 4cP$$

5. *Viscosity in kg/(m s)*

$$Since 1\ Pa = 1\ kg/(m\ s^2),$$

$$\mu = 0.004\ kg/(m\ s)$$

6. *Viscosity in mPa s*

$$Since 1\ mPa\ s = 1\ cP$$

$$\mu = 4\ mPa\ s$$

Example 2.2

Determine the dynamic and kinematic viscosity of air and water at 20°C and 60°C.

Given

Temperature of water = 20°C

Approach

We will obtain values of absolute and kinematic viscosity from Tables A.4.1 and A.4.2.

Solution

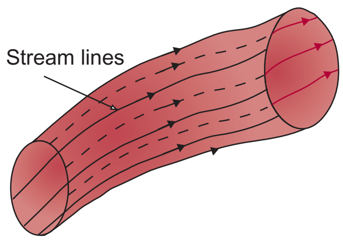
1. *From Table A.4.1 for water at 20°C*
   a. *Dynamic viscosity* $= 993.414 \times 10^{-6}$ *Pa s*
   b. *Kinematic viscosity* $= 1.006 \times 10^{-6}$ m^2/s
2. *From Table A.4.1 for water at 60°C*
   a. *Dynamic viscosity* $= 471.650 \times 10^{-6}$ *Pa s*
   b. *Kinematic viscosity* $= 0.478 \times 10^{-6}$ m^2/s
3. *From Table A.4.2 for air at 20°C*
   a. *Dynamic viscosity* $= 18.240 \times 10^{-6}$ *Pa s*
   b. *Kinematic viscosity* $= 15.7 \times 10^{-6}$ m^2/s
4. *From Table A.4.2 for air at 60°C*
   a. *Dynamic viscosity* $= 19.907 \times 10^{-6}$ *Pa s*
   b. *Kinematic viscosity* $= 19.4 \times 10^{-6}$ m^2/s

As seen from these results, with increasing temperature the dynamic viscosity of water decreases, whereas for gases it increases. The influence of temperature on viscosity of liquids is more pronounced than for gases. The dynamic viscosity of air is much less than that of water.

2.3 HANDLING SYSTEMS FOR NEWTONIAN LIQUIDS

In a food processing plant, liquid foods are processed in a variety of ways, such as by heating, cooling, concentrating, or mixing. The transport of liquid foods from one processing equipment to another is achieved mostly by using pumps, although gravity systems are used when feasible. Depending on the velocity of the liquid and the internal viscous and inertial forces, different types of flow characteristics are obtained. The energy required to pump a liquid will be different under different flow conditions. In this section, we will look at quantitative methods to describe flow characteristics of liquid foods.

In subsequent sections, we will refer to fluid flow along a stream line. At any instant of time, we may consider an imaginary curve in



■ Figure 2.11 Stream lines forming a stream tube. Flow occurs only along stream lines, not across them.

the fluid, called a **stream line**, along which fluid moves (Fig. 2.11). No fluid movement occurs across this curve. The velocity of fluid at any point along the stream line is in a tangential direction along the line. When bunched together into a stream tube, stream lines provide a good indication of the instantaneous flow of a fluid.

2.3.1 The Continuity Equation

The principle of conservation of matter is frequently used to solve problems related to fluid flow. To understand this important principle, consider a fluid flowing in a pipeline, as shown in Figure 2.12. Since the fluid is moving, suppose that in time step δt, the fluid occupying space XX′ moves to space YY′. The distance between X and Y is δx_1 and between X′ and Y′ is δx_2. The cross-sectional area at X is dA_1, and at X′ it is dA_2. We have purposely selected different cross-sectional areas at the two ends to show that our derivation will be applicable to such variations. For the matter to be conserved, the mass contained in space XX′ must be equal to that in YY′. We also note that the fluid contained in space YX′ is common to both initial and final space. Therefore, the mass of fluid in space XY must equal that in space X′Y′. Therefore,

$$\rho_1 A_1 \delta x_1 = \rho_2 A_2 \delta x_2 \qquad (2.12)$$

Dividing by the time step, δt,

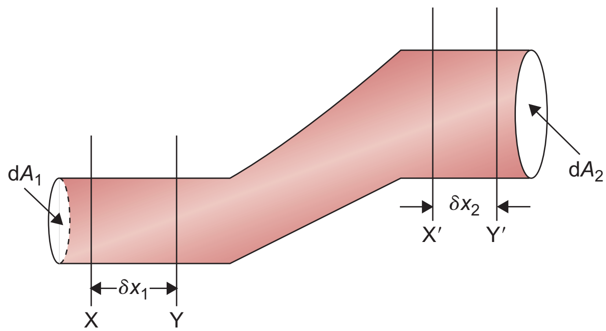
$$\rho_1 A_1 \frac{\delta x_1}{\delta t} = \rho_2 A_2 \frac{\delta x_2}{\delta t} \qquad (2.13)$$

or,

$$\rho_1 A_1 \overline{u}_1 = \rho_2 A_2 \overline{u}_2 \qquad (2.14)$$

where $\overline{u}$ is the average velocity.

■ Figure 2.12 Fluid flow through a pipe of varying cross-section.



Equation (2.14) is the Equation of Continuity. We can express this equation on the basis of either mass flow or volumetric flow rate. In Equation (2.14)

$$\rho A \bar{u} = \dot{m} \tag{2.15}$$

where $\dot{m}$ is the mass flow rate, kg/s. The mass flow rate is a function of density ρ, the cross-sectional area A of the pipe or tube, and the mean velocity $\bar{u}$ of the fluid. Equation (2.15) shows that the mass flow rate remains constant under steady state conditions.

For an incompressible fluid, such as for liquids, the density remains constant. Then, from Equation (2.14)

$$A_1 \bar{u}_1 = A_2 \bar{u}_2 \tag{2.16}$$

where

$$A \bar{u} = \dot{V} \tag{2.17}$$

The volumetric flow rate $\dot{V}$ is a product of cross-sectional area A of the pipe and the mean fluid velocity, $\bar{u}$. According to Equation (2.17), under steady state conditions, the volumetric flow rate remains constant.

The preceding mathematical development will be valid only if we use mean velocity, $\bar{u}$, for the given cross-section. The use of the bar on symbol $\bar{u}$ indicates that it represents a mean value for velocity. We will observe later in Section 2.3.4 that the velocity distribution of a fully developed flow in a pipe is, in fact, parabolic in shape. At this stage, we need to ensure that only mean velocity is selected whenever Equation (2.15) is used.

Example 2.3

The volumetric flow rate of beer flowing in a pipe is 1.8 L/s. The inside diameter of the pipe is 3 cm. The density of beer is 1100 kg/m^3. Calculate the average velocity of beer and its mass flow rate in kg/s. What is the mass flow rate? If another pipe with a diameter of 1.5 cm is used, what will be the velocity for the same volumetric flow rate?

Given

Pipe diameter = 3 cm = 0.03 m
Volumetric flow rate = 1.8 L/s = 0.0018 m^3/s
Density = 1100 kg/m^3

Approach

First, we will calculate velocity, $\bar{u}$, from the given volumetric flow rate using Equation (2.17). Then we will use Equation (2.15) to obtain mass flow rate.

Solution

1. *From Equation (2.17),*

$$\text{Velocity}, \bar{u} = \frac{0.0018\,[m^3/s]}{\dfrac{\pi \times 0.03^2}{4}[m^2]} = 2.55\ m/s$$

2. *From Equation (2.15),*

$$\text{Mass flow rate} = \dot{m} = 1100\,[kg/m^3] \times \frac{\pi \times 0.03^2}{4}[m^2] \times 2.55\,[m/s]$$

$$\dot{m} = 1.98\ kg/s$$

3. *New velocity if the diameter of the pipe is halved and the volumetric flow rate is kept the same:*

$$\bar{u} = \frac{0.0018\,[m^3/s]}{\dfrac{\pi \times 0.015^2}{4}[m^2]} = 10.19\ m/s$$

4. *Note that by halving the diameter, the velocity is increased fourfold.*

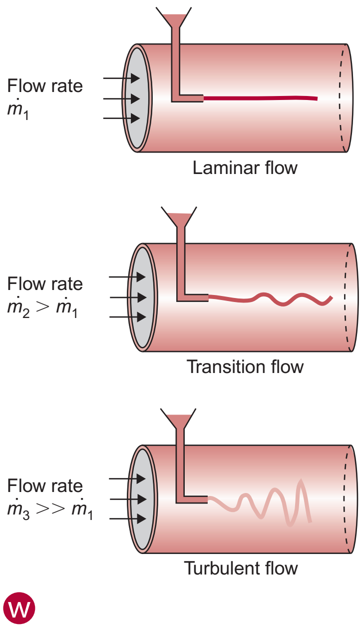


Figure 2.13 Laminar, transitional, and turbulent flow in a pipe.

2.3.2 Reynolds Number

We can conduct a simple experiment to visualize the flow characteristics of a liquid by carefully injecting a dye into the liquid flowing in a pipe. At low flow rates, the dye moves in a straight-line manner in the axial direction, as shown in Figure 2.13. As the flow rate increases to some intermediate level, the dye begins to blur at some distance away from the injection point. The blurring of the dye is caused by movement of some of the dye in the radial direction. At high flow rates, the dye becomes blurred immediately upon injection. At these high flow rates, the dye spreads in a random manner along both the radial and axial direction. The straight-line flow observed at low flow rates is called **laminar** flow; at intermediate flow rates, the flow is called **transitional** flow; and the erratic flow obtained at higher flow rates is called **turbulent** flow.

The flow characteristics for laminar flow are influenced by liquid properties, flow rate, and the dimensions of liquid–solid interfaces. As the mass flow rate is increased, the forces of momentum or inertia increase; but these forces are resisted by friction or viscous forces within the flowing liquid. As these opposing forces reach a certain balance, changes in the flow characteristics occur. Based on experiments conducted by Reynolds (1874)[1], the inertial forces are a function of liquid density, ρ, tube diameter, D, and average fluid velocity, $\bar{u}$. The viscous forces, on the other hand, are a function of liquid viscosity. A dimensionless number, called a Reynolds number, is defined as the ratio of the inertial to the viscous forces:

$$N_{\mathrm{Re}} = \frac{\text{inertial forces}}{\text{viscous forces}} \tag{2.18}$$

or

$$N_{\mathrm{Re}} = \frac{\rho \bar{u} D}{\mu} \tag{2.19}$$

If instead of average velocity, the mass flow rate, $\dot{m}$, is measured or given, then substituting Equation (2.15) in Equation (2.19) and rearranging terms, we obtain

$$N_{\mathrm{Re}} = \frac{4\dot{m}}{\mu \pi D} \tag{2.20}$$

A Reynolds number is most useful in quantitatively describing the flow characteristics of a fluid flowing either in a pipe or on the surfaces of objects of different shapes. We no longer need to limit ourselves to qualitative descriptions of flow such as low, intermediate, or high. Instead, we can use a Reynolds number to specifically identify how a given liquid would behave under selected flow conditions.

The Reynolds number provides an insight into energy dissipation caused by viscous effects. From Equation (2.18), when the viscous forces have a dominant effect on energy dissipation, the Reynolds number is small, or flow is in a laminar region. As long as the Reynolds number is 2100 or less, the flow characteristics are laminar or stream line. A Reynolds number between 2100 and 4000 signifies a transitional flow. A Reynolds number greater than 4000 indicates turbulent flow denoting small influence of viscous forces on energy dissipation.

---

[1] Osborne Reynolds (1842–1912) was a British physicist, engineer, and educator. He was appointed the first professor of engineering at Owens College, Manchester, where he retired in the same position in 1905. His major work was in the study of hydrodynamics. He developed a theory of lubrication, studied the condensation process, and provided a mathematical foundation (in 1883) to the turbulence phenomenon in fluid flow. His work resulted in important redesign of boilers and condensers, and development of turbines.

Example 2.4

A 3-cm inside diameter pipe is being used to pump liquid food into a buffer tank. The tank is 1.5 m diameter and 3 m high. The density of the liquid is 1040 kg/m^3 and viscosity is 1600×10^{-6} Pa s.

a. What is the minimum time to fill the tank with this liquid food if it is flowing under laminar conditions in the pipe?

b. What will be the maximum time to fill the tank if the flow in the pipe is turbulent?

Given

Pipe diameter = 3 cm = 0.03 m
Tank height = 3 m
Tank diameter = 1.5 m
Density of liquid = 1040 kg/m^3
Viscosity of liquid = 1600×10^{-6} Pa s = 1600×10^{-6} kg/m s

Approach

For (a), we will use the maximum Reynolds number in the laminar range of 2100 and calculate the flow rate. For (b), we will use a minimum Reynolds number in the turbulent region of 4000 and calculate the flow rate. The time to fill the tank will be obtained from the volume of the tank and the volumetric flow rate.

Solution

Part (a)

1. *From Equation (2.19), maximum velocity under laminar conditions is*

$$\bar{u} = \frac{2100\,\mu}{\rho D} = \frac{2100 \times 1600 \times 10^{-6}\,[\text{kg/ms}]}{1040\,[\text{kg/m}^3] \times 0.03\,[\text{m}]} = 0.108\ \text{m/s}$$

Then, volumetric flow rate using the pipe cross-sectional area and Equation (2.17) is

$$\dot{V} = \frac{\pi \times 0.03^2\,[\text{m}^2]}{4} \times 0.108\,[\text{m/s}] = 7.63 \times 10^{-5}\ \text{m}^3/\text{s}$$

2. *Volume of tank*

$$\text{Volume of tank} = \frac{\pi(\text{diameter})^2(\text{height})}{4}$$

$$= \frac{\pi \times 1.5^2\,[\text{m}^2] \times 3\,[\text{m}]}{4}$$

$$= 5.3\ \text{m}^3$$

3. *The minimum time to fill the tank = (volume of tank)/(volumetric flow rate)*

$$= \frac{5.3\,[m^3]}{7.63 \times 10^{-5}\,[m^3/s]} = 6.95 \times 10^4\ s = 19.29\ h$$

Part (b)

4. *From Equation (2.19), minimum velocity under turbulent flow conditions is*

$$\bar{u} = \frac{4000\,\mu}{\rho D} = \frac{4000 \times 1600 \times 10^{-6}\,[kg/ms]}{1040\,[kg/m^3] \times 0.03\,[m]} = 0.205\ m/s$$

Then, volumetric flow rate using the pipe cross-sectional area and Equation (2.17) is

$$\dot{V} = \frac{\pi \times 0.03^3\,[m^2]}{4} \times 0.20\,[m/s] = 1.449 \times 10^{-4}\ m^3/s$$

5. *The maximum time to fill the tank = (volume of tank)/(volumetric flow rate)*

$$= \frac{5.3\,[m^3]}{1.449 \times 10^{-4}\,[m^3/s]} = 3.66 \times 10^4\ s = 10.16\ h$$

6. *The minimum time to fill the tank under laminar conditions is 19.29 h, whereas the maximum time to fill the tank under turbulent conditions is 10.16 h.*

Example 2.5

At what velocity does air and water flow convert from laminar to transitional in a 5 cm diameter pipe at 20°C?

Given

Pipe diameter = 5 cm = 0.05 m
Temperature = 20°C
From Table A.4.1 for water,
Density = 998.2 kg/m^3
Viscosity = 993.414 × 10^{-6} Pa s
From Table A.4.4 for air,
Density = 1.164 kg/m^3
Viscosity = 18.240 × 10^{-6} Pa s

Approach

We will use a Reynolds number of 2100 for change from laminar to transitional flow.

Solution

1. *From the Reynolds number and Equation (2.19), we obtain velocity as*

$$\bar{u} = \frac{N_{Re}\mu}{\rho D}$$

2. *For water*

$$\bar{u} = \frac{2100 \times 993.414 \times 10^{-6}\,[kg/ms]}{998.2\,[kg/m^3] \times 0.05\,[m]}$$

$$\bar{u} = 0.042\ m/s$$

3. *For air*

$$\bar{u} = \frac{2100 \times 18.240 \times 10^{-6}\,[kg/ms]}{1.164\,[kg/m^3] \times 0.05\,[m]}$$

$$\bar{u} = 0.658\ m/s$$

4. *To change from laminar to transitional flow, the calculated velocities of air and water for a 5-cm pipe are quite low; typically, much higher fluid velocities are used in commercial practice. Therefore, the fluid flow in industry is generally in the transition or turbulent region. Mostly, we encounter laminar flow in the case of more viscous liquids.*

2.3.3 Entrance Region and Fully Developed Flow

When a liquid enters a pipe, there is a certain initial length of the pipe, called the entrance region, where the flow characteristics of the liquid are quite different from those in the following length of pipe. As shown in Figure 2.14, immediately at the entrance to the pipe, the liquid has a uniform velocity profile, identified by the same length of arrows in the diagram. As it begins to move into the pipe, the liquid next to the inside wall of the pipe is held back by friction between the liquid and the wall surface. The velocity of the liquid is zero at the wall and increases toward the central axis of the pipe. Therefore, the boundary (or the wall surface) begins to influence the velocity profile. As shown in Figure 2.14, in the entrance region, the boundary layer develops from location X to Y. At location Y, the effect

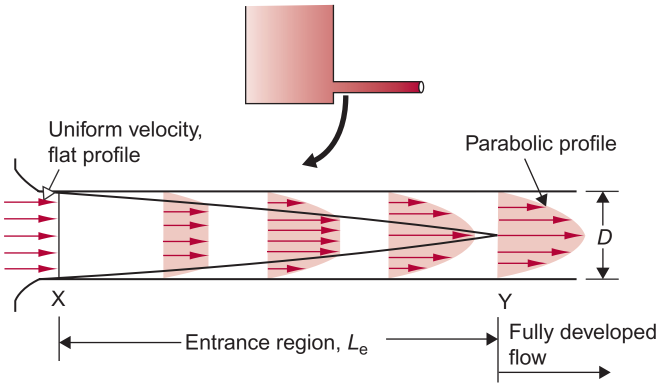


Figure 2.14 Velocity profile in a fluid flowing in a pipe.

of the boundary layer on the velocity profile extends all the way to the central axis. The cross-sectional velocity profile at Y is parabolic in shape (as we will mathematically derive in the following section). From X to Y, the region is called the **entrance region**, and the liquid flow in the region beyond Y is commonly referred to as **fully developed flow**.

Using dimensional analysis, it has been shown that the dimensionless entrance length, L_e/D, is a function of the Reynolds number. Therefore, the entrance length, L_e may be calculated from the following expressions:

For laminar flow,

$$\frac{L_e}{D} = 0.06\, N_{Re} \tag{2.21}$$

and for turbulent flow,

$$\frac{L_e}{D} = 4.4(N_{Re})^{1/6} \tag{2.22}$$

Example 2.6

A 2-cm diameter pipe is 10 m long and delivers wine at a rate of 40 L/min at 20°C. What fraction of the pipe represents the entrance region?

Given

Pipe diameter = 2 cm = 0.02 m
Length = 10 m
Flow rate = 40 L/min = 6.67×10^{-4} m^3/s
Temperature = 20°C

Approach

Since the properties of wine are not given, we will approximate properties of wine as those of water obtained from Table A.4.2. First, we will determine the Reynolds number and then select an appropriate equation to calculate the entrance region from Equations (2.21) and (2.22).

Solution

1. *The average velocity is obtained from Equation (2.17)*

$$\bar{u} = \frac{0.000667\,[m^3/s]}{\dfrac{\pi \times 0.02^2}{4}\,[m^2]}$$

$$= 2.12\ m/s$$

2. *The Reynolds number, using Equation (2.19), is*

$$N_{Re} = \frac{998.2\,[kg/m^3] \times 2.12\,[m/s] \times 0.02\,[m]}{993.414 \times 10^{-6}\,[Pa\ s]}$$

$$N_{Re} = 42{,}604$$

Therefore, the flow is turbulent, and we select Equation (2.22) to determine the entrance region.

3. *Using Equation (2.22)*

$$L_e = 0.02\,[m] \times 4.4 \times (42{,}604)^{1/6}$$

$$L_e = 0.52\ m$$

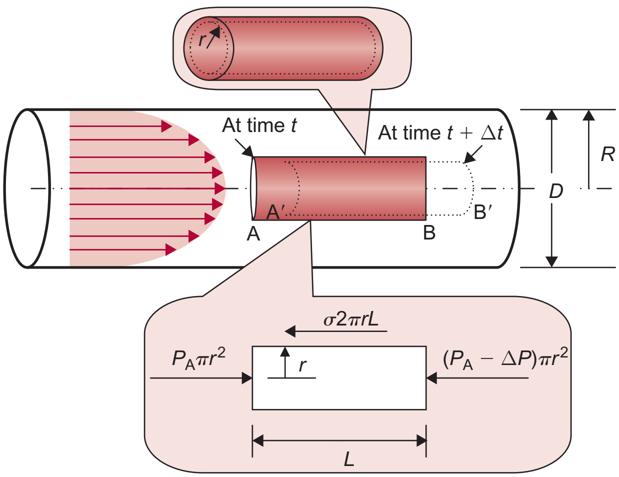
4. *The entrance region is 5% of the total length of the pipe.*

2.3.4 Velocity Profile in a Liquid Flowing Under Fully Developed Flow Conditions

Calculations to determine the velocity profile in a pipe depend upon whether the region of interest is at the entrance or further along the pipe where the flow is fully developed. In the entrance region, the calculations are complicated because the liquid velocity depends not only on the radial distance from the pipe centerline, r, but also on the axial distance from the entrance, x. On the other hand, the velocity profile in the fully developed region depends only on the radial distance from the central axis, *r*. In the entrance region, three forces—gravitational, pressure, and inertial—influence the flow. Largely due to inertial forces, the flow in the entrance region accelerates, and the velocity profile changes from location X to Y, as shown in Figure 2.14. A mathematical description of flow in the entrance region and under turbulent conditions is highly complex and beyond the scope of this book. Therefore, we will determine the velocity profile in a liquid flowing in a straight horizontal pipe of a constant diameter under fully developed laminar flow conditions.

Consider fluid flow taking place under steady and fully developed conditions in a constant-diameter pipe. Forces due to pressure and gravity cause the fluid flow in the fully developed region. In the case of a horizontal pipe, gravitational effects are negligible. Therefore, for the purpose of this analysis, we will consider only forces due to pressure. When a viscous liquid (a liquid with viscosity greater than zero) flows in a pipe, the viscous forces within the liquid oppose the pressure forces. Application of pressure is therefore necessary for flow to occur, as it overcomes the viscous forces opposing the flow. Furthermore, the flow takes place without accelerating; the velocity profile within the fully developed flow region does not change with location along the x-axis. For the flow to be steady, a balance must exist between the pressure and viscous forces in the liquid. We will conduct a force balance to analyze this case.

Using Newton's second law of motion, we can describe forces acting on a small element of liquid as shown in Figure 2.15. The cylindrical



■ **Figure 2.15** Force balance for a liquid flowing in a pipe.

element is of radius r and length L. The pipe diameter is D. Initially at time t, the location of the element is identified from A to B. After a small lapse of time Δt, the liquid element moves to a new location A′B′. The ends of the element, A′B′, indicating velocity profile, are shown distorted, indicating that under fully developed flow conditions, the velocity at the central axis is maximum and decreases with increasing r.

Since the pipe is horizontal, we neglect the gravitational forces. The pressure varies from one axial location to another, but remains constant along any vertical cross-section of the pipe. Let us assume that the pressure on the cross-sectional face at A is P_A and at B is P_B. If the decrease of pressure from A to B is ΔP, then $\Delta P = P_A - P_B$.

As seen in the force diagram for Figure 2.15, the pressure forces acting on the liquid element are as follows:

On the vertical cross-sectional area, πr^2,

$$\text{at location A, pressure forces} = P_A \pi r^2 \tag{2.23}$$

$$\text{at location B, pressure forces} = (P_A - \Delta P)\pi r^2 \tag{2.24}$$

and on the circumferential area, $2\pi r L$,

$$\text{forces opposing pressure forces due to viscous effects} = \sigma 2\pi r L \tag{2.25}$$

where σ is the shear stress.

According to Newton's second law of motion, the force in the x direction, $F_x = ma_x$. As noted earlier in this section, under fully developed flow conditions, there is no acceleration, or $a_x = 0$. Therefore, $F_x = 0$. Thus, for the liquid element, all forces acting on it must balance, or,

$$P_A \pi r^2 - (P_A - \Delta P)\pi r^2 - \sigma 2\pi r L = 0 \tag{2.26}$$

or simplifying,

$$\frac{\Delta P}{L} = \frac{2\sigma}{r} \tag{2.27}$$

For Newtonian liquids, the shear stress is related to viscosity as seen earlier in Equation (2.10). For pipe flow, we rewrite this equation in cylindrical coordinates as

$$\sigma = -\mu \frac{\mathrm{d}u}{\mathrm{d}r} \tag{2.28}$$

Note that $\mathrm{d}u/\mathrm{d}r$ is negative in case of a pipe flow, the velocity decreasing with increasing radial distance, r, as seen in Figure 2.15.

Therefore, we have introduced a negative sign in Equation (2.28) so that we obtain a positive value for shear stress, σ. Substituting Equation (2.27) in Equation (2.28),

$$\frac{du}{dr} = -\left(\frac{\Delta P}{2\mu L}\right) r \tag{2.29}$$

integrating,

$$\int du = -\frac{\Delta P}{2\mu L}\int r dr \tag{2.30}$$

or

$$u(r) = -\left(\frac{\Delta P}{4\mu L}\right) r^2 + C_1 \tag{2.31}$$

where C_1 is a constant.

For a viscous fluid flowing in a pipe, $u = 0$ at $r = R$; therefore

$$C_1 = \frac{\Delta P}{4\mu L} R^2 \tag{2.32}$$

Therefore, the velocity profile for a laminar, fully developed flow, in a horizontal pipe, is:

$$u(r) = \frac{\Delta P}{4\mu L}(R^2 - r^2) \tag{2.33}$$

or

$$u(r) = \frac{\Delta P R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{2.34}$$

Equation (2.34) is an equation of a parabola. Therefore, for fully developed flow conditions we obtain a parabolic velocity profile. Furthermore, from Equation (2.34), substituting $r = 0$, the maximum velocity, u_{max}, is obtained at the pipe centerline, or

$$u_{max} = \frac{\Delta P R^2}{4\mu L} \tag{2.35}$$

Next, let us determine the volumetric flow rate by integrating the velocity profile across the cross-section of the pipe. First, we will examine a small ring of thickness dr with an area dA, where $dA = 2\pi r dr$, as shown in Figure 2.15. The velocity, u, in this thin annular ring is assumed to be constant. Then the volumetric flow rate through the annular ring, $\dot{V}_{ring}$ is:

$$\dot{V}_{ring} = u(r)dA = u(r)2\pi r dr \tag{2.36}$$

The volumetric flow rate for the entire pipe cross-section is obtained by integration as follows:

$$\dot{V} = \int u(r)\mathrm{d}A = \int_{r=0}^{r=R} u(r)2\pi r\mathrm{d}r \tag{2.37}$$

or, substituting Equation (2.34) in (2.37),

$$\dot{V} = \frac{2\pi\Delta PR^2}{4\mu L}\int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\mathrm{d}r \tag{2.38}$$

or

$$\dot{V} = \frac{\pi R^4 \Delta P}{8\mu L} \tag{2.39}$$

The mean velocity, $\bar{u}$, is defined as the volumetric flow rate divided by the cross-sectional area of the pipe, πR^2, or

$$\bar{u} = \frac{\dot{V}}{\pi R^2} \tag{2.40}$$

or, substituting Equation (2.39) in Equation (2.40), we obtain

$$\bar{u} = \frac{\Delta PR^2}{8\mu L} \tag{2.41}$$

Equation (2.39) is called Poiseuille's Law. The flow characteristics of fully developed laminar flow were described independently in experiments by two scientists, G. Hagen in 1839 and J. Poiseuille[2] in 1840.

If we divide Equation (2.41) by (2.35), we obtain

$$\frac{\bar{u}}{u_{max}} = 0.5 \quad \text{(Laminar Flow)} \tag{2.42}$$

From Equation (2.42), we observe that the average velocity is half the maximum velocity for fully developed laminar flow conditions. Furthermore, the radius (or diameter) of the pipe has a dramatic influence on the flow rate, as seen in Equation (2.39). Doubling the diameter increases the volumetric flow rate 16-fold.

---

[2] Jean-Louis-Marie Poiseuille (1799–1869), a French physiologist, studied the flow rate of fluids under laminar condition in circular tubes. The same mathematical expression also was determined by Gotthilf Hagen; therefore the relationship is called the Hagen–Poiseuille equation. Poiseuille also studied the circulation of blood and the flow of fluids in narrow tubes.

For turbulent flow in the fully developed region, the mathematical analysis necessary to obtain a velocity profile is complex. Therefore, the following empirical expression is generally used.

$$\frac{\overline{u}(r)}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/j} \tag{2.43}$$

where j is a function of the Reynolds number. For most applications $j = 7$ is recommended. Under turbulent conditions, the velocity profile may be obtained from

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/7} \tag{2.44}$$

Equation (2.44) is also called the Blassius 1/7th power law.

A volumetric flow rate under turbulent conditions may be obtained in a similar manner as for laminar conditions. Substituting Equation (2.43) in Equation (2.37),

$$\dot{V} = \int_{r=0}^{r=R} u_{max}\left(1 - \frac{r}{R}\right)^{1/j} 2\pi r \mathrm{d}r \tag{2.45}$$

Integrating Equation (2.45), we get

$$\dot{V} = 2\pi u_{max} \frac{R^2 j^2}{(j+1)(2j+1)} \tag{2.46}$$

A relationship between average and maximum velocity may be obtained by substituting Equation (2.40) in Equation (2.46), and we get,

$$\frac{\overline{u}}{u_{max}} = \frac{2j^2}{(j+1)(2j+1)} \tag{2.47}$$

Substituting the value 7 for j in Equation (2.47),

$$\frac{\overline{u}}{u_{max}} = 0.82 \quad \text{(Turbulent Flow)} \tag{2.48}$$

Thus, in the case of turbulent flow, the average velocity is 82% of the maximum velocity. The maximum velocity occurs at the central axis of the pipe.

Example 2.7

A fluid is flowing under laminar conditions in a cylindrical pipe of 2 cm diameter. The pressure drop is 330 Pa, the viscosity of the fluid is 5 Pa s, and the pipe is 300 cm long. Calculate the mean velocity and velocity of fluid at different radial locations in the pipe.

Given

Diameter of pipe = 2 cm
Length of pipe = 300 cm
Pressure drop = 330 Pa
Viscosity = 5 Pa s

Approach

We will use Equation (2.33) to calculate velocity at different radial locations.

Solution

1. *From Equation (2.33)*

$$u = \frac{\Delta P}{4\mu L}(R^2 - r^2)$$

The velocity is calculated at r = 0, 0.25, 0.5, 0.75, and 1 cm.

r = 0 cm	*u = 0.055 cm/s*
r = 0.25 cm	*u = 0.0516 cm/s*
r = 0.5 cm	*u = 0.0413 cm/s*
r = 0.75 cm	*u = 0.0241 cm/s*
r = 1 cm	*u = 0 cm/s*

2. *The mean velocity is calculated as 0.0275 cm/s; this value is half the maximum velocity.*

2.3.5 Forces Due to Friction

The forces that must be overcome in order to pump a liquid through a pipe derive from several sources. As we saw in Section 2.2.3, viscous forces are important in liquid flow; these forces occur due to the movement of one layer over another. The other important forces are due to friction between the liquid and the surface of the wall of a pipe. When a fluid flows through a pipe, some of its mechanical energy is dissipated due to friction. It is common to refer to this dissipated energy as a frictional energy loss. Though truly not a loss, some of the mechanical energy supplied to the liquid to cause flow is actually converted into heat, therefore all the mechanical energy is not available as useful energy in the liquid transport system.

The friction forces vary with conditions such as flow rates, as described with a Reynolds number, and surface roughness. The influence of the friction forces is expressed in the form of a **friction factor** *f*. The following mathematical development is for laminar flow conditions.

The friction factor is the ratio between the shear stress at the wall, σ_w, to the kinetic energy of the fluid per unit volume.

$$f = \frac{\sigma_w}{\rho \bar{u}^2/2} \tag{2.49}$$

Rewriting Equation (2.27) for shear stress at the wall, $r = D/2$

$$\sigma_w = \frac{D\Delta P}{4L} \tag{2.50}$$

Substituting Equation (2.50) in Equation (2.49) we obtain

$$f = \frac{\Delta PD}{2L\rho \bar{u}^2} \tag{2.51}$$

By rearranging terms in Equation (2.41), the pressure drop in fully developed laminar flow conditions is determined as

$$\Delta P = \frac{32\mu \bar{u} L}{D^2} \tag{2.52}$$

Substituting Equation (2.52) in Equation (2.51), we obtain

$$f = \frac{16}{N_{Re}} \tag{2.53}$$

where f is called the Fanning friction factor. Note that many civil and mechanical engineering textbooks refer to a different friction factor, called Darcy[3] friction factor, with the same symbol, f. The Darcy friction factor is four times the Fanning friction factor. In chemical engineering literature, the Fanning friction factor is used more commonly, and in this text we will use only the Fanning friction factor.

The previous calculations leading to Fanning friction factor are for laminar flow conditions only. In cases of transitional and turbulent flow conditions, the mathematical derivations become highly complex. For situations involving flow conditions other than laminar flow, we will use a graphical chart that presents friction factor as a function of the Reynolds number. This chart, called a **Moody chart**, is shown in Figure 2.16. The Moody chart presents the friction factor as a function of the Reynolds number for various magnitudes of relative roughness of the pipes. At a low Reynolds number ($N_{Re} \ll 2100$), the curve is described by Equation (2.53) and is not influenced by surface

[3] Henri-Philibert-Gaspard Darcy (1803–1858), a French hydraulic engineer, was the first person to develop a mathematical formulation of laminar flow of fluids in porous materials. His work laid the foundation for the subject of ground-water hydrology. In Dijon, his native city, he supervised the design and construction of the municipal water supply system. In his work he studied the flow of groundwater through granular material.

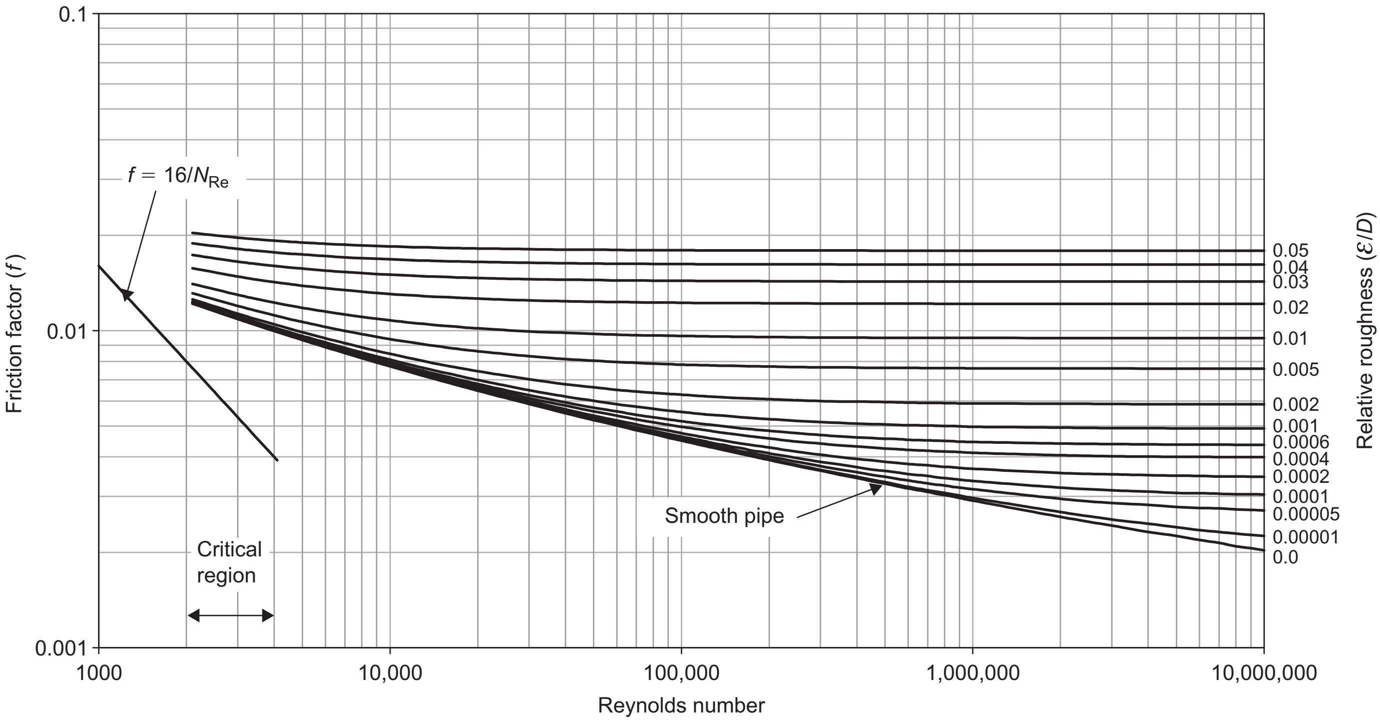


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Figure 2.16 The Moody diagram for the Fanning friction factor. Equivalent roughness for new pipes (ε in meters): cast iron, 259×10^{-6}; drawn tubing, 1.5235×10^{-6}, galvanized iron, 152×10^{-6}; steel or wrought iron, 45.7×10^{-6}. (Based on L.F. Moody, 1944. Trans *ASME*, *66*, 671.)

roughness, ε, of the pipe. In the transition from laminar to turbulent flow or critical region, either set of curves can be used. Most often, the friction factor is selected for turbulent flow, since it ensures that the pressure loss due to friction will not be underestimated. The Moody chart is accurate to ±15 percent.

From the Moody chart, it is evident that the friction factor is never zero, even for smooth pipes. Because there is always some roughness at the microscopic level, a fluid will stick to the pipe surface regardless of how smooth it is. Thus, there is always a certain frictional loss when a fluid flows in a pipe.

An explicit equation to estimate the friction factor, *f*, was proposed by Haaland (1983). A slightly modified form of this equation for the Darcy friction factor, Equation (2.54) is recommended for calculating the Fanning friction factor for a turbulent region, preferably with the use of a spreadsheet.

$$\frac{1}{\sqrt{f}} \approx -3.6 \log\left[\frac{6.9}{N_{\mathrm{Re}}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \tag{2.54}$$

Example 2.8

Water at 30°C is being pumped through a 30-m section of 2.5 cm diameter steel pipe at a mass flow rate of 2 kg/s. Compute the pressure loss due to friction in the pipe section.

Given

Density (ρ) = 995.7 kg/m^3, *from Table A.4.1*
Viscosity (μ) = 792.377×10^{-6} *Pa s, from Table A.4.1*
Length (L) of pipe = 30 m
Diameter (D) of pipe = 2.5 cm = 0.025 m
Mass flow rate ($\dot{m}$) = 2 kg/s

Approach

The pressure drop due to friction is computed using Equation (2.51) with the information given. Equation (2.51) requires knowledge of the friction factor f as obtained from Figure 2.16. Figure 2.16 can be used once the turbulence (N_{Re}) *and relative roughness* (ε/D) *values have been determined*

Solution

1. *Compute mean velocity* $\bar{u}$ *from Equation (2.15):*

$$\bar{u} = \frac{(2\ \mathrm{kg/s})}{(995.7\ \mathrm{kg/m^3})[\pi(0.025\ \mathrm{m})^2/4]} = 4.092\ \mathrm{m/s}$$

2. *Compute the Reynolds number:*

$$N_{Re} = \frac{(995.7\ kg/m^3)(0.025\ m)(4.092\ m/s)}{(792.377 \times 10^{-6}\ Pa\ s)} = 128{,}550$$

3. *Using the given information and Figure 2.16, relative roughness can be computed:*

$$\varepsilon/D = \frac{45.7 \times 10^{-6}\ m}{0.025\ m} = 1.828 \times 10^{-3}$$

4. *Using the computed Reynolds number and the computed relative roughness, friction factor f is obtained from Figure 2.16:*

$$f = 0.006$$

5. *Using Equation (2.51):*

$$\frac{\Delta P}{\rho} = 2(0.006)\frac{(4.092\ m/s)^2(30\ m)}{(0.025\ m)} = 241.12\ m^2/s^2$$

6. *Note that ($1\ J = 1\ kg\ m^2/s^2$)*

$$\frac{\Delta P}{\rho} = 241.12\ m^2/s^2 = 241.12\ J/kg$$

represents the energy consumed due to friction on a per-unit-mass basis.

7. *The pressure loss is calculated as (Note that $1\ J = 1\ kg\ m^2/s^2$)*

$$\Delta P = (241.12\ J/kg)(995.7\ kg/m^3) = 240.80 \times 10^3\ kg/(m\ s^2)$$

$$\Delta P = 240.08\ kPa.$$

2.4 FORCE BALANCE ON A FLUID ELEMENT FLOWING IN A PIPE—DERIVATION OF BERNOULLI EQUATION

As noted in this chapter, fluids begin to move when a nonzero resultant force acts upon them. The resultant force brings about a change in the momentum of the fluid. We recall from physics that momentum is the product of mass and velocity. Under steady-flow conditions, the resultant force acting on a liquid must equal the net rate of change in momentum. We will use these concepts to derive one of the most widely used equations in fluid flow, called the Bernoulli equation.

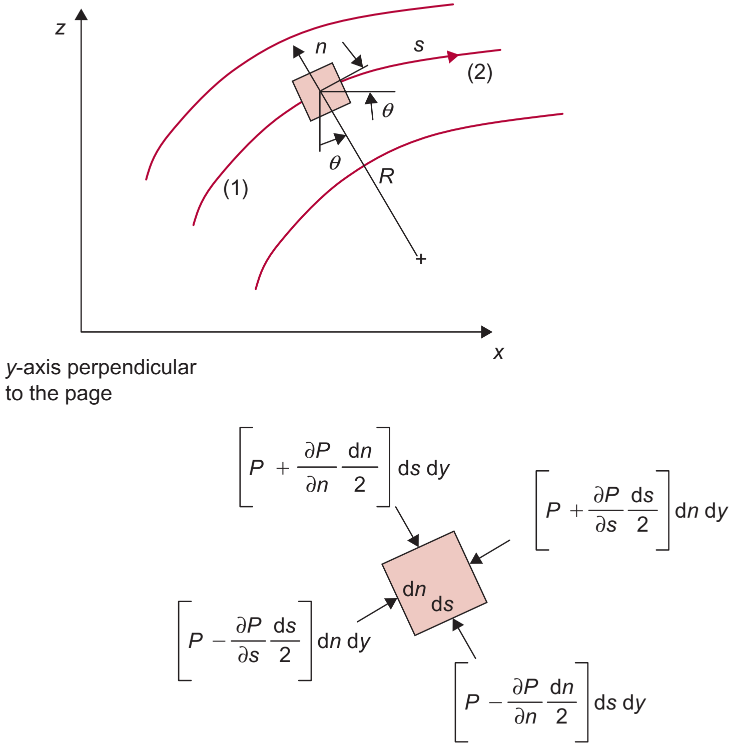


Figure 2.17 Force balance on a small volume of fluid.

Let us consider a particle of fluid moving along a stream line from location (1) to (2) as shown in Figure 2.17. The flow is assumed to be steady and the liquid has a constant density. The fluid is inviscid, meaning that its viscosity is zero. The x and z axes are shown; the y direction is in the perpendicular direction from the x–z horizontal plane. The s-direction is along the stream line and n-direction is normal to the s-direction. The velocity of the particle is u.

The forces acting on the particle, neglecting any forces due to friction, are the result of particle weight and pressure. Let us consider these forces separately.

a. A force component of the particle weight exerted in the s-direction is obtained as follows. The volume of the particle is $\mathrm{d}n\ \mathrm{d}s\ \mathrm{d}y$. If the density of the liquid is ρ, then,

$$\text{weight of the particle} = \rho g\ \mathrm{d}n\ \mathrm{d}s\ \mathrm{d}y \tag{2.55}$$

and,

$$\text{the component of force in the } s\text{-direction due to weight} = -\rho g \sin\theta \, \mathrm{d}n \, \mathrm{d}s \, \mathrm{d}y$$

since,

$$\sin\theta = \frac{\partial z}{\partial s}$$

force component in the s-direction

$$\text{due to weight} = -\rho g \frac{\partial z}{\partial s} \mathrm{d}n \, \mathrm{d}s \, \mathrm{d}y \qquad (2.56)$$

b. The second force acting on the particle due to pressure in the s-direction, as seen in Figure 2.17, is as follows.

$$\text{pressure force on particle} = \left(P - \frac{\partial P}{\partial s}\frac{\mathrm{d}s}{2}\right) \mathrm{d}n \, \mathrm{d}y - \left(P + \frac{\partial P}{\partial s}\frac{\mathrm{d}s}{2}\right) \mathrm{d}n \, \mathrm{d}y \qquad (2.57)$$

Or, canceling and rearranging terms in the equation:

$$\text{pressure force on particle in } s\text{-direction} = -\frac{\partial P}{\partial s} \mathrm{d}n \, \mathrm{d}y \, \mathrm{d}s \qquad (2.58)$$

from Equations (2.56) and (2.58), the resultant force acting on the particle

$$\text{in the } s\text{-direction} = \left(-\frac{\partial P}{\partial s} - \rho g \frac{\partial z}{\partial s}\right) \mathrm{d}n \mathrm{d}s \mathrm{d}y$$

or,

$$\text{resultant force per unit volume} = -\frac{\partial P}{\partial s} - \rho g \frac{\partial z}{\partial s} \qquad (2.59)$$

Because of this resultant force, the particle will accelerate as it moves along the stream line. Therefore, the velocity will change from u to $u + (\partial u/\partial s)\mathrm{d}s$ as the particle moves from s to $s + \mathrm{d}s$.

Recall from physics that momentum is mass multiplied with velocity. The rate of change of momentum of the particle due to the action of resultant force is then

$$\rho\left(\frac{u+\frac{\partial u}{\partial s}\mathrm{d}s-u}{\mathrm{d}t}\right)$$

or simply

$$\text{rate of change of momentum} = \rho\frac{\partial u}{\partial s}\frac{\mathrm{d}s}{\mathrm{d}t} \tag{2.60}$$

But

$$\frac{\mathrm{d}s}{\mathrm{d}t} = u$$

Then,

$$\text{rate of change of momentum} = \rho u\frac{\partial u}{\partial s} \tag{2.61}$$

The resultant force must equal the rate of change of momentum of the particle, or from Equation (2.59) and Equation (2.61):

$$\frac{\partial P}{\partial s} + \rho u\frac{\partial u}{\partial s} + \rho g\frac{\partial z}{\partial s} = 0 \tag{2.62}$$

This equation is also called the Euler equation of motion. If we multiply both sides by ds, we get

$$\frac{\partial P}{\partial s}\mathrm{d}s + \rho u\frac{\partial u}{\partial s}\mathrm{d}s + \rho g\frac{\partial z}{\partial s}\mathrm{d}s = 0 \tag{2.63}$$

The first term in Equation (2.63) expresses the change in pressure along the stream line; the second term is a change in velocity, and the third term is a change in elevation. Using laws of calculus, we can simply write this equation as

$$\frac{\mathrm{d}P}{\rho} + u\mathrm{d}u + g\mathrm{d}z = 0 \tag{2.64}$$

Integrating the preceding equation from location (1) to (2) along the stream line, we obtain

$$\int_{P_1}^{P_2}\frac{\mathrm{d}P}{\rho} + \int_{u_1}^{u_2}u\mathrm{d}u + g\int_{z_1}^{z_2}\mathrm{d}z = 0 \tag{2.65}$$

or, evaluating limits and multiplying by ρ, and rearranging,

$$P_1 + \frac{1}{2}\rho u_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho u_2^2 + \rho g z_2 = \text{constant} \tag{2.66}$$

Equation (2.66) is called the **Bernoulli equation**, named after a Swiss mathematician, Daniel Bernoulli. It is one of the equations most widely used to solve problems in fluid dynamics. The application of this equation to many problems involving fluid flow provides great insight. However, if the assumptions used in deriving the equation are not followed, then erroneous results are likely to be obtained. Again, note the main assumptions used in deriving this equation:

- Locations 1 and 2 are on the same stream line.
- The fluid has a constant density, therefore the fluid is incompressible.
- The flow is inviscid; that is, the viscosity of the fluid is zero.
- The flow is steady.
- No shaft work is done on or by the fluid.
- No heat transfer takes place between the fluid and its surroundings.

As we will observe in some of the examples in this section, the Bernoulli equation may provide reasonable approximations even if the assumptions are not strictly followed. For example, fluids with low viscosities may approximate inviscid conditions.

Another frequently used form of the Bernoulli equation is written in terms of "head." If we divide Equation (2.66) by specific weight of the fluid, ρg, we obtain

$$\underbrace{\frac{P}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{u^2}{2g}}_{\text{velocity head}} + \underbrace{z}_{\text{elevation head}} = \text{constant} = h_{\text{total}} \tag{2.67}$$

Each term on the left-hand side of Equation (2.67) is expressed in units of length, m. The three terms are pressure head, velocity head, and elevation head, respectively. The sum of these three heads is a constant, called total head, h_{total}. The total head of a fluid flowing in a pipe is measured with a Pitot-tube at the stagnation point, as we will discuss in Section 2.7.1. The use of the Bernoulli equation is illustrated in Examples 2.9 and 2.10.

Example 2.9

A 3-m diameter stainless steel tank contains wine. The tank is filled to 5 m depth. A discharge port, 10 cm in diameter, is opened to drain the wine. Calculate the discharge velocity of wine, assuming the flow is steady and frictionless, and the time required in emptying it.

Given

Height of tank = 5 m
Diameter of tank = 3 m

Approach

We will use the Bernoulli equation using the assumptions of steady and frictionless flow.

Solution

1. *We select location (1) as the wine free surface, and location (2) as the nozzle exit. The pressure at (1) is atmospheric. The velocity at location (1) is low enough for us to consider it as a quasi-steady state condition with zero velocity.*
2. *In the Bernoulli equation, Equation (2.66), $P_1 = P_2 = P_{atm}$, $\rho_1 = \rho_2$ and $\bar{u}_1 = 0$; therefore,*

$$gz_1 = \frac{1}{2}\bar{u}_2^2 + gz_2$$

or

$$\bar{u}_2 = \sqrt{2g(z_2 - z_1)}$$

This formula is named after Evangelista Torricelli, who proposed it in 1644.

3. *Substituting the known values in the Torricelli formula,*

$$\bar{u} = \sqrt{2 \times 9.81\,[m/s^2] \times 5\,[m]} = 9.9\,m/s$$

Then, volumetric flow rate from the discharge port, using Equation (2.17),

$$= \frac{\pi \times 0.10^2\,[m^2]}{4} \times 9.9\,[m/s] = 0.078\,m^3/s$$

4. *The volume of the tank is*

$$\frac{\pi \times 3^2\,[m^2]}{4} \times 5\,[m] = 35.3\,m^3$$

5. *Time to empty the tank equals*

$$\frac{35.3[m^3]}{0.078[m^3/s]} = 452.6\,s = 7.5\,min$$

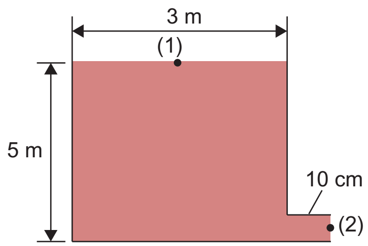


Figure E2.1 Discharge from a tank, for conditions given in Example 2.9.

Example 2.10

A 1.5 cm diameter tube is being used to siphon water out of a tank. The discharge end of the siphon tube is 3 m below the bottom of the tank. The water level in the tank is 4 m. Calculate the maximum height of the hill over which the tube can siphon the water without cavitation. The temperature of the water is 30°C.

Given

Siphon tube diameter = 1.5 cm = 0.015 m
Height of water in tank = 4 m
Discharge location below the bottom of tank = 3 m
Temperature of water = 30°C

Approach

Assuming an inviscid, steady, and incompressible flow, we will apply the Bernoulli equation at locations (1), (2), and (3). We will then calculate the pressure at location (2) and compare it with the vapor pressure of water at 30°C. Note that the atmospheric pressure is 101.3 kPa.

Solution

1. To apply the Bernoulli equation at locations (1), (2), and (3), we note that

$$P_1 = P_3 = P_{atm}, \quad \bar{u}_1 = 0, \quad z_1 = 4 \text{ m}, \quad z_3 = -3 \text{ m}.$$

2. From the equation of continuity, Equation (2.16)

$$A_2\bar{u}_2 = A_3\bar{u}_3$$

Therefore

$$\bar{u}_2 = \bar{u}_3$$

3. Applying the Bernoulli equation, Equation (2.66), between locations (1) and (3) gives

$$\rho g z_1 = \frac{1}{2}\rho\bar{u}_3^2 + \rho g z_3$$

$$\bar{u}_3 = \sqrt{2g(z_1 - z_3)}$$

then

$$\bar{u}_3 = \sqrt{2 \times 9.81\,[\text{m/s}^2] \times (4 - (-3))\,[\text{m}]}$$

$\bar{u}_3 = 11.72$ m/s, same as at location (2), or, $\bar{u}_2 = 11.72$ m/s.

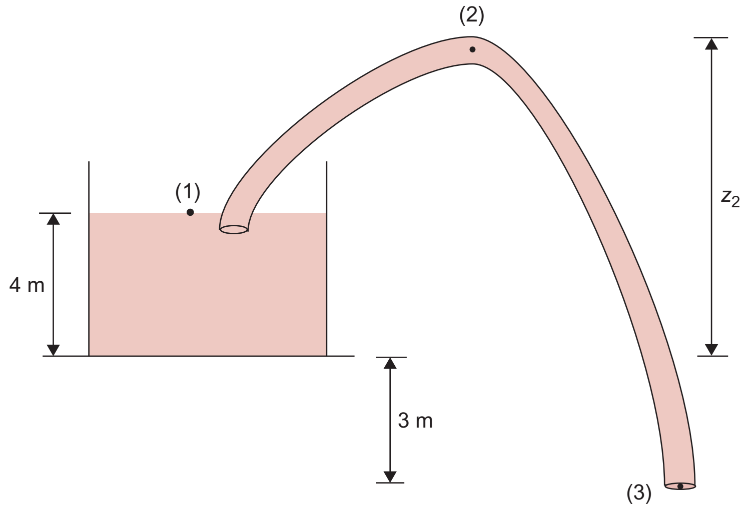


Figure E2.2 Siphoning water out of a tank, for conditions given in Example 2.10.

4. *From Table A.4.2, at 30°C, the vapor pressure of water = 4.246 kPa. Again using the Bernoulli equation between locations (1) and (2), noting that $\bar{u}_1 = 0$, we get*

$$z_2 = z_1 + \frac{P_1}{\rho g} - \frac{P_2}{\rho g} - \frac{\bar{u}_2^2}{2g}$$

Substituting values and noting that 1 Pa = 1 kg/(ms²)

$$z_2 = 4[m] + \frac{(101.325 - 4.246) \times 1000[Pa]}{995.7[kg/m^3] \times 9.81\ [m/s^2]}$$

$$-\frac{1}{2} \times (11.72)^2 \left[\frac{m^2}{s^2}\right] \times \frac{1}{9.81}\left[\frac{s^2}{m}\right]$$

$$z_2 = 6.93\ m$$

5. *If z_2 is more than 6.93 m then cavitation will occur. If the discharge end of the siphon tube is lowered, say 5 m below the bottom of the tank, then the velocity will be higher and the value of z_2 will be lower.*

2.5 ENERGY EQUATION FOR STEADY FLOW OF FLUIDS

As noted in preceding sections, fluid flow occurs with the application of a force. Thus, a fluid transport system depends on a source of energy; for liquids we use pumps, whereas for gases we use blowers.

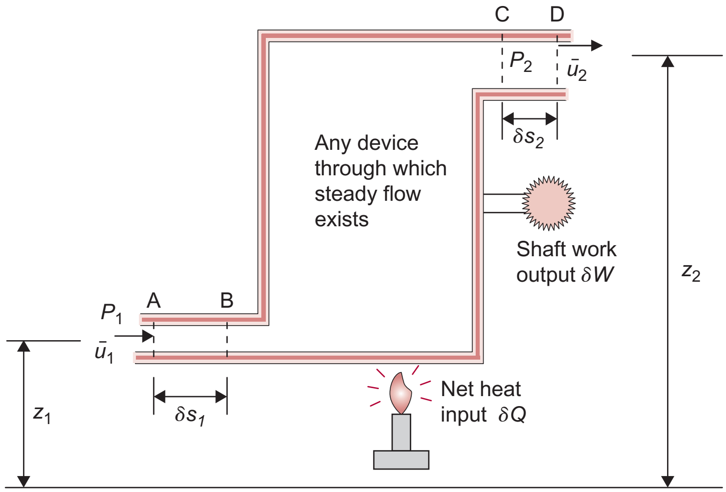


Figure 2.18 A device with a steady flow.

In this section, we will develop mathematical expressions useful for determining the energy requirements for fluid flow. This mathematical development requires the application of the first law of thermo-dynamics and concepts presented in Chapter 1.

Consider a system involving fluid flow, such as the one shown in Figure 2.18. We assume that (a) the flow is continuous and under steady-state conditions, and the mass flow rate entering and exiting the system is constant; (b) fluid properties and conditions between inlet and outlet do not vary; (c) heat and shaft work between the fluid and the surroundings are transferred at a constant rate; and (d) energy transfer due to electricity, magnetism, and surface tension are negligible.

For the flow system shown in Figure 2.18, in a unit time, a constant amount of heat, δQ, is added to the system, and the system does a constant rate of work, δW, on the surroundings (e.g., it could rotate a shaft if it were a turbine or a steam engine; but if work is done by the surroundings on the fluid, as by a pump, then rate of work will carry a negative sign). At the inlet, the fluid velocity is $\overline{u}_1$, pressure P_1, and elevation z_1. At the exit, the fluid velocity is $\overline{u}_2$, pressure P_2, and elevation z_2. At any moment, a certain packet of fluid is between locations A and C. After a short time duration, δt, this fluid packet moves to locations B and D. According to the Continuity equation, Equation (2.14), the mass of liquid, δm, in AB is the same as in CD. At the inlet, on a per unit mass basis, the fluid has a specific internal energy, E'_{i1}, kinetic energy, $\frac{1}{2}\overline{u}_1^2$, and potential energy, gz_1. Let the energy embodied in the system between B and C be E_{B-C}. Therefore, the energy embodied in the fluid between A and C is

$$E_{A-C} = E_{A-B} + E_{B-C} \tag{2.68}$$

or,

$$E_{A-C} = \delta m\left(E'_{i1} + \frac{1}{2}\overline{u}_1^2 + gz_1\right) + E_{B-C} \tag{2.69}$$

After a short time duration, δt, as the fluid packet moves from A–C to B–D, the energy embodied in the fluid between B and D will be

$$E_{B-D} = E_{B-C} + E_{C-D} \tag{2.70}$$

or,

$$E_{B-D} = E_{B-C} + \delta m\left(E'_{i2} + \frac{1}{2}\overline{u}_2^2 + gz_2\right) \tag{2.71}$$

Therefore, the increase in energy for the selected packet of fluid as it moves from A–C to B–D is

$$\delta E_{\text{increase}} = E_{B-D} - E_{A-C} \tag{2.72}$$

$$\begin{aligned}\delta E_{\text{increase}} = &\left[E_{B-C} + \delta m\left(E'_{i2} + \frac{1}{2}\overline{u}_2^2 + gz_2\right)\right] \\ &- \left[\delta m\left(E'_{i1} + \frac{1}{2}\overline{u}_1^2 + gz_1\right) + E_{B-C}\right]\end{aligned} \tag{2.73}$$

or simplifying,

$$\delta E_{\text{increase}} = \delta m\left[(E'_{i2} - E'_{i1}) + \frac{1}{2}(\overline{u}_2^2 - \overline{u}_1^2) + g(z_2 - z_1)\right] \tag{2.74}$$

During the time interval δt, when the liquid packet moves from A–C to B–D, the work done *by* the flowing fluid *on* the surroundings is δW. (Note that it will be $-\delta W$ if we were using a pump that does work on the fluid.) The heat transfer into the fluid system is δQ. Furthermore, there is work associated with pressure forces (as noted in Section 2.4). The work done by the fluid at the exit is $P_2 A_2 \delta x_2$, and at the inlet, the work done on the fluid is $-P_1A_1\delta x_1$, where areas A_1 and A_2 are cross-sectional areas at the inlet and exit, and P_1 and P_2 are pressures at the inlet and exit. Therefore, total work done by the flowing fluid is

$$\delta W_{\text{total}} = \delta W + P_2A_2\delta x_2 - P_1A_1\delta x_1 \qquad (2.75)$$

From the energy balance, we note that the change in energy of the system is the heat added minus the total work done by the fluid on the surroundings, or,

$$\delta E_{\text{increase}} = \delta Q - \delta W_{\text{total}} \qquad (2.76)$$

Substituting Equation (2.74) and Equation (2.75) in Equation (2.76), and rearranging, we obtain,

$$\delta Q = \delta m\left[(E'_{i2} - E'_{i1}) + \frac{1}{2}(\bar{u}_2^2 - \bar{u}_1^2) + g(z_2 - z_1)\right] + \delta W + P_2A_2\delta x_2 - P_1A_1\delta x_1 \qquad (2.77)$$

From the mass balance, we know

$$\delta m = \rho_1A_1\delta x_1 = \rho_2A_2\delta x_2 \qquad (2.78)$$

Dividing Equation (2.77) by δm and substituting Equation (2.78),

$$\frac{\delta Q}{\delta m} = (E'_{i2} - E'_{i1}) + \frac{1}{2}(\bar{u}_2^2 - \bar{u}_1^2) + g(z_2 - z_1) + \frac{\delta W}{\delta m} + \frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} \qquad (2.79)$$

Rearranging terms,

$$Q_m = \left(\frac{P_2}{\rho_2} + \frac{1}{2}\bar{u}_2^2 + gz_2\right) - \left(\frac{P_1}{\rho_1} + \frac{1}{2}\bar{u}_1^2 + gz_1\right) + (E'_{i2} - E'_{i1}) + W_m \qquad (2.80)$$

Equation (2.80) is the general energy equation for a system involving steady fluid flow, where Q_m is the heat added to the fluid system per unit mass, and W_m is the work done per unit mass by the fluid system on its surroundings (such as by a turbine).

Note that for an incompressible and inviscid fluid (viscosity = 0) if no transfer of heat or work takes place ($W_m = 0$, $Q_m = 0$) and the internal energy of the flowing fluid remains constant, then Equation (2.80) reduces to the Bernoulli equation, as presented in Section 2.4.

However, in case of a real fluid, we cannot ignore its viscosity. A certain amount of work is done to overcome viscous forces, commonly referred to as fluid friction. Due to frictional work, there is a transfer of energy into heat, with an increase in temperature. However, the temperature rise is usually very small and of little practical value, and the frictional work is often referred to as a loss of useful energy. Therefore, in Equation (2.80), we may express the terms $(E'_{i2} - E'_{i1})$ as E_f, the frictional loss of energy. Furthermore, in problems involving pumping of a liquid we may replace W_m with the work done by the pump, E_p. Note a change in sign will be required, since W_m was work done by a fluid on the surroundings. Assuming no transfer of heat with the surroundings, or $Q_m = 0$, Equation (2.80) is rewritten as:

$$\frac{P_2}{\rho_2} + \frac{1}{2}\bar{u}_2^2 + gz_2 + E_f = \frac{P_1}{\rho_1} + \frac{1}{2}\bar{u}_1^2 + gz_1 + Ep \tag{2.81}$$

Rearranging terms in Equation (2.81) to obtain an expression for the energy requirements of a pump, E_p, per unit mass, and noting that for an incompressible fluid, $\rho_2 = \rho_1 = \rho$,

$$E_p = \frac{P_2 - P_1}{\rho} + \frac{1}{2}(\bar{u}_2^2 - \bar{u}_1^2) + g(z_2 - z_1) + E_f \tag{2.82}$$

Equation (2.82) contains terms for pressure energy, kinetic energy, potential energy, and energy loss associated with frictional forces, respectively. We will now consider these items individually, to note any necessary modifications and practical implications.

2.5.1 Pressure Energy

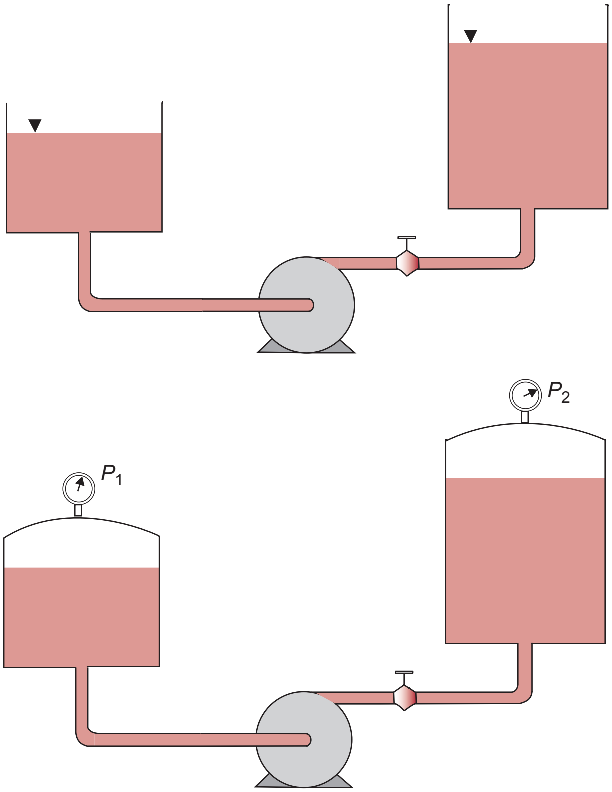
The first term on the right-hand side of Equation (2.82) denotes energy dissipation related to the change in pressure between locations (1) and (2). If the transport system (Fig. 2.19a) connects two tanks, both of which are exposed to the atmosphere, then there is no change in pressure, or $P_1 - P_2 = 0$. However, in situations where one or both tanks are under pressure or vacuum (Fig. 2.19b), the pressure difference needs to be accounted for. Changes in pressure may add to energy requirements as

$$\frac{\Delta P}{\rho} = \frac{P_2 - P_1}{\rho} \tag{2.83}$$

Note that liquid density does not change in the type of systems being analyzed. In other words, the flow is incompressible.

2.5.2 Kinetic Energy

The second term in the right-hand side of Equation (2.82) accounts for the change in velocity of the flowing fluid from location (1) to (2),



■ **Figure 2.19** Pumping liquid between two tanks.

resulting in a change in kinetic energy. In deriving the energy equation (2.82), we assumed that the fluid velocity is uniform across the entire cross-section. However, due to viscous effects, the velocity is never uniform across the pipe cross-section, but varies as we observed in Section 2.3.4. Therefore, we must use a correction factor, α, and modify the kinetic energy term in Equation (2.82) as follows:

$$\text{Kinetic Energy} = \frac{\bar{u}_2^2 - \bar{u}_1^2}{2\alpha} \quad (2.84)$$

where for laminar flow, $\alpha = 0.5$, and for turbulent flow, $\alpha = 1.0$. Note that the units of the kinetic energy term in Equation (2.84) are expressed as J/kg, as follows:

$$\text{Kinetic Energy} = \frac{\bar{u}^2}{2} \equiv \frac{\text{m}^2}{\text{s}^2} \equiv \frac{\text{kg m}^2}{\text{kg s}^2} \equiv \left(\frac{\text{kg m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{N m}}{\text{kg}} \equiv \frac{\text{J}}{\text{kg}}$$

2.5.3 Potential Energy

The energy required to overcome a change in elevation during liquid transport is potential energy. The general expression for change in potential energy per unit mass is

$$= g(z_2 - z_1) \tag{2.85}$$

where z_2 and z_1 are the elevations indicated in Figure 2.18, and the acceleration due to gravity (g) converts the elevation to energy units (J/kg).

$$\text{Potential Energy} = gz \equiv \frac{\text{m}^2}{\text{s}^2} \equiv \frac{\text{kg m}^2}{\text{kg s}^2} \equiv \left(\frac{\text{kg m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{N m}}{\text{kg}} \equiv \frac{\text{J}}{\text{kg}}$$

2.5.4 Frictional Energy Loss

The frictional energy loss for a liquid flowing in a pipe is composed of major and minor losses. Or,

$$E_f = E_{\text{f,major}} + E_{\text{f,minor}} \tag{2.86}$$

The **major losses**, $E_{\text{f,major}}$ are due to the flow of viscous liquid in the straight portions of a pipe. Equation (2.51) may be rearranged to give an expression for pressure drop per unit density to account for energy loss due to friction per unit mass, $E_{\text{f,major}}$, as

$$E_{\text{f,major}} = \frac{\Delta P}{\rho} = 2f\frac{\overline{u}^2 L}{D} \tag{2.87}$$

where f is the friction factor obtained from the Moody diagram or Equation (2.54).

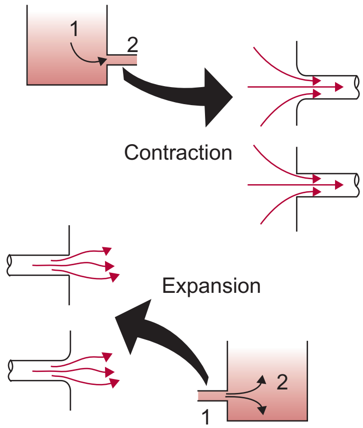
The second type of frictional losses, the **minor losses**, $E_{\text{f,minor}}$, are due to various components used in pipeline systems—such as valves, tees, and elbows—and to contraction of fluid when it enters from a tank into a pipe or expansion of a fluid when it empties out from a pipe into a tank. Although these types of losses are called minor losses, they can be quite significant. For example, if a valve installed in a pipeline is fully closed, then it offers an infinite resistance to flow, and the loss is certainly not minor. The minor losses have three components:

$$E_{\text{f,minor}} = E_{\text{f,contraction}} + E_{\text{f,expansion}} + E_{\text{f,fittings}} \tag{2.88}$$

We will consider each of these components separately.

2.5.4.1 *Energy Loss Due to Sudden Contraction, $E_{f,contraction}$*

When the diameter of a pipe suddenly decreases or, in a limiting case, when a liquid held in a tank enters a pipe, there is a contraction



■ Figure 2.20 Liquid flow through a contraction and an expansion.

in flow (Fig. 2.20). A sudden contraction in the cross-section of the pipe causes energy dissipation. If $\bar{u}$ is the upstream velocity, the energy loss due to sudden contraction is evaluated as

$$\frac{\Delta P}{\rho} = C_{fc}\frac{\bar{u}^2}{2} \tag{2.89}$$

where,

$$C_{fc} = 0.4\left[1.25 - \left(\frac{A_2}{A_1}\right)\right] \quad \text{where } \frac{A_2}{A_1} < 0.715$$
$$C_{fc} = 0.75\left[1 - \left(\frac{A_2}{A_1}\right)\right] \quad \text{where } \frac{A_2}{A_1} > 0.715 \tag{2.90}$$

A limiting case of sudden contraction is when a pipe is connected to a large reservoir. As seen in Figure 2.20, for this case, the diameter A_1 is much larger than A_2, therefore, $A_2/A_1 = 0$ and $C_{fc} = 0.5$.

2.5.4.2 *Energy Loss Due to Sudden Expansion*

In a similar manner to sudden contraction, a sudden increase in the cross-section of a pipe will contribute to energy loss due to friction. The energy loss is

$$\frac{\Delta P}{\rho} = C_{fe}\frac{\bar{u}^2}{2} \tag{2.91}$$

and coefficient, C_{fe}, in this case is,

$$C_{fe} = \left(1 - \frac{A_1}{A_2}\right)^2 \tag{2.92}$$

where parameters having a subscript of one are located upstream from the expansion joint. For the limiting case, when a pipe exits into a reservoir, A_2 is much larger than A_1, and $A_1/A_2 = 0$, and $C_{fe} = 1.0$.

2.5.4.3 *Energy Losses Due to Pipe Fittings*

All pipe fittings such as elbows, tees, and valves will contribute to energy losses due to friction. The energy loss associated with pipe fittings is

$$\frac{\Delta P}{\rho} = C_{ff}\frac{\bar{u}^2}{2} \tag{2.93}$$

Table 2.2 Friction Losses for Standard Fittings

Type of Fitting	Cff
Elbows	
Long radius 45°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 90°, flanged	0.2
Regular 45°, threaded	0.4
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
180° Return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
Tees	
Branch flow, flanged	1.0
Branch flow, threaded	2.0
Line flow, flanged	0.2
Line flow, threaded	0.9
Union threaded	0.8
Valves	
Angle, fully open	2
Ball valve, ⅓ closed	5.5
Ball valve, ⅔ closed	210
Ball valve, fully open	0.05
Diaphragm valve, open	2.3
Diaphragm valve, ¼ closed	2.6
Diaphragm valve, ½ closed	4.3
Gate, ¾ closed	17
Gate, ¼ closed	0.26
Gate, ½ closed	2.1
Gate, fully open	0.15
Globe, fully open	10
Swing check, backward flow	∞
Swing check, forward flow	2

Typical values of loss coefficient, C_{ff}, for various fittings are given in Table 2.2. Depending upon how many fittings are used in a fluid transport system, the C_{ff} is the summed up value for all the fittings in Equation (2.93). We will illustrate this procedure in Example 2.11.

Other processing equipment that may be installed in the liquid transport system, such as a heat exchanger, will usually have some assigned pressure drop due to friction. If not, a value for the pressure drop should be obtained by measurement. The measured pressure drop value is then divided by the liquid density to obtain the appropriate energy units.

2.5.5 Power Requirements of a Pump

We can compute the power requirements of a pump by knowing all the changes in energy associated with pumping liquid from one location to another. The energy requirements for pumping a liquid may be expressed by expanding Equation (2.82), as follows:

$$E_p = \frac{P_2 - P_1}{\rho} + \frac{1}{2}(\bar{u}_2^2 - \bar{u}_1^2) + g(z_2 - z_1) + E_{f,\text{major}} + E_{f,\text{minor}} \tag{2.94}$$

or

$$\begin{aligned} E_p = {} & \frac{P_2 - P_1}{\rho} + \frac{1}{2}(\bar{u}_2^2 - \bar{u}_1^2) + g(z_2 - z_1) \\ & + \frac{2f\bar{u}^2 L}{D} + C_{fe}\frac{\bar{u}^2}{2} + C_{fc}\frac{\bar{u}^2}{2} + C_{ff}\frac{\bar{u}^2}{2} \end{aligned} \tag{2.95}$$

or, dividing each term by g, we may determine the pump requirements in terms of head, as

$$\begin{aligned} h_{\text{pump}} = {} & \underbrace{\frac{P_2 - P_1}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{1}{2g}(\bar{u}_2^2 - \bar{u}_1^2)}_{\text{velocity head}} + \underbrace{(z_2 - z_1)}_{\text{elevation head}} + \underbrace{\frac{2f\bar{u}^2 L}{gD}}_{\text{major losses head}} \\ & + \underbrace{C_{fe}\frac{\bar{u}^2}{2g} + C_{fc}\frac{\bar{u}^2}{2g} + C_{ff}\frac{\bar{u}^2}{2g}}_{\text{minor losses head}} \end{aligned} \tag{2.96}$$

We can compute the power requirements for the pump, Φ, by noting that power is the rate of doing work; if the mass flow rate $\dot{m}$ is known, then

$$\text{Power} = \Phi = \dot{m}(E_P) \tag{2.97}$$

where E_p is the work done per unit mass by the pump on the fluid, as given by Equation (2.95).

To calculate pump sizes, we need to incorporate accurate sizes of the pipes being used into computations. The information in Table 2.3 provides the type of values needed for this purpose. Note the variations in diameters of steel pipe as compared with sanitary pipe for the same nominal size.

Table 2.3 Pipe and Heat-Exchanger Tube Dimensions

	Steel pipe (Schedule 40)		Sanitary pipe		Heat-exchanger tube (18 gauge)	
Nominal size (in)	**ID in/(m)**	**OD in/(m)**	**ID in/(m)**	**OD in/(m)**	**ID in/(m)**	**OD in/(m)**
0.5	0.622 (0.01579)[a]	0.840 (0.02134)	–	–	0.402 (0.01021)	0.50 (0.0127)
0.75	0.824 (0.02093)	1.050 (0.02667)			0.652 (0.01656)	0.75 (0.01905)
1	1.049 (0.02644)	1.315 (0.03340)	0.902 (0.02291)	1.00 (0.0254)	0.902 (0.02291)	1.00 (0.0254)
1.5	1.610 (0.04089)	1.900 (0.04826)	1.402 (0.03561)	1.50 (0.0381)	1.402 (0.03561)	1.50 (0.0381)
2.0	2.067 (0.0525)	2.375 (0.06033)	1.870 (0.04749)	2.00 (0.0508)	–	–
2.5	2.469 (0.06271)	2.875 (0.07302)	2.370 (0.06019)	2.5 (0.0635)		
3.0	3.068 (0.07793)	3.500 (0.08890)	2.870 (0.07289)	3.0 (0.0762)	–	–
4.0	4.026 (0.10226)	4.500 (0.11430)	3.834 (0.09739)	4.0 (0.1016)		

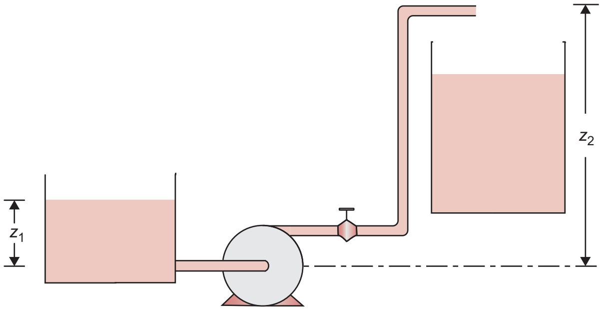
Source: Toledo (1991)
[a]Numbers in parentheses represent the dimension in meters.

Example 2.11

A 20° Brix (20% sucrose by weight) apple juice is being pumped at 27°C from an open tank through a 1-in nominal diameter sanitary pipe to a second tank at a higher level, illustrated in Figure E2.3. The mass flow rate is 1 kg/s through 30 m of straight pipe with two 90° standard elbows and one angle valve. The supply tank maintains a liquid level of 3 m, and the apple juice leaves the system at an elevation of 12 m above the floor. Compute the power requirements of the pump.

Given

Product viscosity (μ) = 2.1×10^{-3} *Pa s, assumed to be the same as for water, from Table A.2.4*
Product density (ρ) = *997.1 kg/m*^{3}*, estimated from the density of water at 25°C*
Pipe diameter (D) = 1 in nominal = 0.02291 m, from Table 2.3
Mass flow rate ($\dot{m}$) = *1 kg/s*



■ **Figure E2.3** Pumping water from one tank to another, for Example 2.11.

Pipe length (L) = 30 m
90° standard elbow friction, from Table 2.2
Angle valve friction, from Table 2.2
Liquid level $z_1 = 3$ m, $z_2 = 12$ m

Approach

Power requirements for the pump can be computed using the mechanical energy balance.

Solution

1. *First, compute mean velocity using mass flow rate equation.*

$$\bar{u} = \frac{\dot{m}}{\rho A} = \frac{(1\ \text{kg/s})}{(997.1\ \text{kg/m}^3)[\pi(0.02291\ \text{m})^2/4]} = 2.433\ \text{m/s}$$

2. *By computation of the Reynolds number,*

$$N_{Re} = \frac{(997.1\ \text{kg/m}^3)(0.02291\ \text{m})(2.433\ \text{m/s})}{(2.1 \times 10^{-3}\ \text{Pa s})}$$
$$= 26{,}465$$

it is established that flow is turbulent.

3. *By using the energy Equation (2.82) and identification of reference points, the following expression is obtained:*

$$g(3) + E_p = g(12) + \frac{(2.433)^2}{2} + E_f$$

where reference 1 is at the upper level of the supply tank, $\bar{u}_1 = 0$, and $P_1 = P_2$.

4. *By computing E_f, the power can be determined. Based on $N_{Re} = 2.6465 \times 10^4$ and smooth pipe, $f = 0.006$ from Figure 2.16.*
5. *The entrance from the tank to the pipeline can be accounted for by Equation (2.90), where*

$$C_{fc} = 0.4(1.25 - 0) \quad \text{since} \quad D_2^2/D_1^2 = 0$$
$$= 0.5$$

and

$$\frac{\Delta P}{\rho} = 0.5\frac{(2.433)^2}{2} = 1.48\ J/kg$$

6. *The contribution of the two elbows and the angle valve to friction is determined by using C_{ff} factors from Table 2.2. C_{ff} for 90° threaded regular elbows is 1.5 and for angle valve, fully open, is 2. Then by using Equation (2.93), we get*

$$\frac{\Delta P}{\rho} = \frac{(2 \times 1.5 + 2) \times 2.433^2}{2} = 14.79$$

For the 30 m length, the friction loss is obtained from Equation (2.87):

$$E_f = \frac{2 \times 0.006 \times 2.433^2 \times 30}{0.02291} = 93.01$$

7. *Then the total friction loss is*

$$E_f = 93.01 + 14.79 + 1.48 = 109.3\ J/kg$$

8. *Using the expression obtained in Equation (2.95),*

$$E_P = 9.81(12 - 3) + \frac{2.433^2}{2} + 109.3$$

$$E_P = 200.5\ J/kg$$

This represents the energy requirements of the pump.

9. *Since power is energy use per unit time,*

$$\text{Power} = (200.5\ J/kg)(1\ kg/s) = 200.5\ J/s$$

10. *This answer must be considered theoretical, since delivery of power to pump may be only 60% efficient; then actual power is*

$$\text{Power} = 200.5/0.6 = 334.2\ W$$

Example 2.12

Develop a spreadsheet using the data given in Example 2.11. Rework the problem using the spreadsheet. Determine the influence on the power requirements of changing the pipe length to 60, 90, 120, and 150 m. Also, determine the influence on the power requirement of changing the pipe diameter to 1.5 in, 2 in, and 2.5 in nominal diameters.

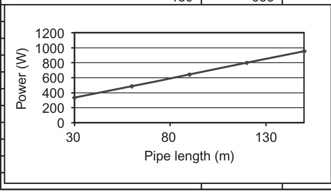
Given

The conditions are the same as in Example 2.11.

Approach

We will develop a spreadsheet using Excel™. The mathematical expressions will be the same as those used in Example 2.11. For the friction factor we will use Equation (2.54).

	A	B	C	D	E	F	G	H
1	Given							
2	Viscosity (Pa s)	0.0021						
3	Density (kg/m^3)	997.1						
4	Diameter (m)	0.02291						
5	Mass flow rate (kg/s)	1						
6	Pipe length (m)	30						
7	C_{ff} Elbows from Table 2.2	1.5						
8	C_{ff} Angle Valve from Table 2.2	2						
9	Low liquid level (m)	3						
10	High liquid level (m)	12						
11								
12	Mean velocity	2.432883	=B5/(B3*PI()*B4^2/4)					
13	Reynolds Number	26464.62	=B3*B12*B4/B2					
14	Entrance Losses	1.47973	=0.5*B12^2/2					
15	Friction factor	0.006008	=(−1/(3.6*LOG(6.9/B13)))^2					
16	Friction Loss_pipe length	93.12623	=2*B15*B6*B12^2/B4					
17	Friction loss_fittings	14.7973	=(2*B7+B8)*B12^2/2					
18	Total Friction loss	109.4033	=B14+B16+B17					
19	Energy for Pump	200.6527	=9.81*(B10−B9)+B12^2/2+B18					
20	Power	334.4212	=B19*B5/0.6					
21								
22	Pipe length	Power		Diameter	Power			
23	30	334		0.02291	334			
24	60	490		0.03561	172			
25	90	645		0.04749	154			
26	120	800		0.06019	149			
27	150	955						



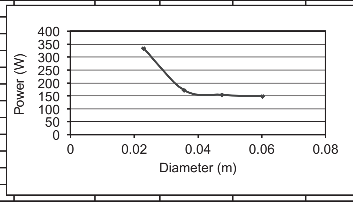


Figure E2.4 A spreadsheet solution for Example 2.12.

Solution

The spreadsheet is developed as shown in Figure E2.4. All mathematical equations are the same *as those used in Example 2.11. The influence of changing length and diameter is seen in the plots. As is evident, for the conditions used in this example, there is a dramatic influence on power requirement when the pipe diameter is decreased from 2.5 in to 1.5 in.*

2.6 PUMP SELECTION AND PERFORMANCE EVALUATION

2.6.1 Centrifugal Pumps

In Section 2.1.2, we described some salient features of different types of pumps. We will now consider centrifugal pumps in greater detail, as these are the most commonly used pumps for pumping water and a variety of low-viscosity Newtonian liquids.

As seen in Figure 2.21, a centrifugal pump has two components: an impeller firmly attached to a rotating shaft, and a volute-shaped housing, called casing, that encloses the impeller. The impeller contains a number of blades, called vanes, that usually are curved backward. The shaft of the

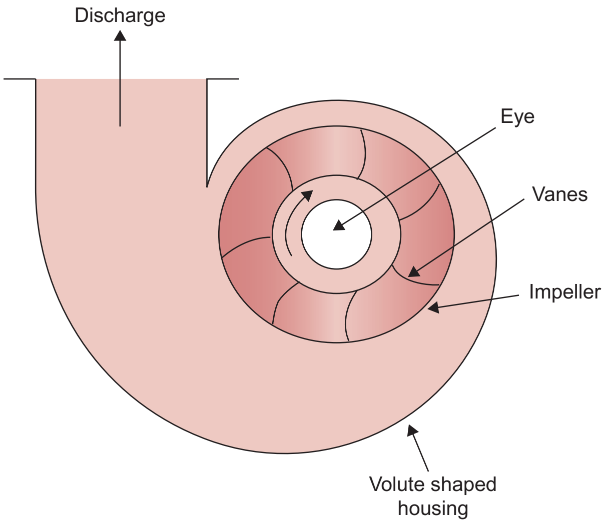


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Figure 2.21 A centrifugal pump.

pump is rotated using either an electric motor or an engine. As the shaft rotates, the impeller, fixed to the shaft, also rotates. The liquid is sucked through an opening in the casing, called the eye. Due to the rotating impeller, and the direction of the vanes, the liquid flows from the eye to the periphery of the impeller. Therefore, work is done on the liquid by the rotating vanes. In moving from the eye to the periphery, the velocity of the liquid increases, raising its kinetic energy. However, as the liquid enters the peripheral zone, or the volute-shaped casing, the liquid velocity decreases. This decrease in velocity occurs because of the increasingly larger area in the volute. The decrease in liquid velocity causes its kinetic energy to decrease, which is converted into an increase in pressure. Thus, the discharging liquid has a higher pressure when compared with the liquid entering the pump at the suction eye. In summary, the main purpose of the pump is to raise the pressure of the liquid as it moves from suction to discharge.

Although considerable theory has been developed regarding the operation of centrifugal pumps, the mathematical complexity prevents theory alone from being an adequate basis for selection of pumps for a given flow system. Therefore, experimental data are usually obtained for actual pump performance. These data, obtained by manufacturers, are supplied with the pump as pump performance curves. The engineer's task is then to select an appropriate pump based on its performance characteristics. In calculations involving fluid flow and pump selection, a commonly used term is the head. We will first develop a general understanding of this term.

2.6.2 Head

In designing pumps, a common term used to express the energy of a fluid is the **head**. As noted previously, in Chapter 1, head is expressed in meters of liquid.

In Equation (2.96) if we sum all the energy terms into head for various items connected to the suction side of the pump, the summed up value of head is called **suction head**. Similarly, on the discharge side, if we convert all the energy terms to head and add them together, we obtain the **discharge head**.

Let us consider a pump being used to lift water from tank A to tank B, as shown in Figure 2.22a. First let us assume that there is no loss in energy due to friction within the pipes or any of the fittings. The height of water in tank A is 5 m from the pump centerline and it is 10 m in tank B. The pressure gauges on the discharge and suction side read 0.49 bar and 0.98 bar, respectively. This is in accordance with Equation (1.20). The total head is then the discharge head minus the suction head, or $10 - 5 = 5$ m.

The same tanks in Figure 2.22a are shown in Figure 2.22b, except the pressure gauge on the suction side reads a pressure of 0.39 bar. This decrease, of 0.1 bar (equivalent to 1.02 m of water) is due to the frictional losses in the pipe and other fittings used to convey the water from tank A to the pump. This is a more realistic situation because there will always be some frictional loss due to flow of a viscous liquid.

In Figure 2.22c, tank A is located below the pump centerline. In this case, first the pump must lift water from a lower level to the pump centerline. This is called the **suction lift**. The total head is then calculated as suction plus the discharge head or $7 + 10 = 17$ m.

The fourth case, Figure 2.22d involves water in a tank on the suction side where it is held under pressure (0.30 bar), as indicated by the gauge in the headspace of the tank. The pressure gauge in the suction line indicates 0.79 bar. The suction head is greater than the actual height of the water because of the applied pressure.

2.6.3 Pump Performance Characteristics

In designing liquid transport systems that involve pumps, two items are necessary: (1) quantitative information about a pump being considered, and (2) the energy requirements associated with liquid flow through various components of the transport system such as pipes, tanks, processing equipment, and fittings. The information on the pump should inform us about the energy that the pump will add to

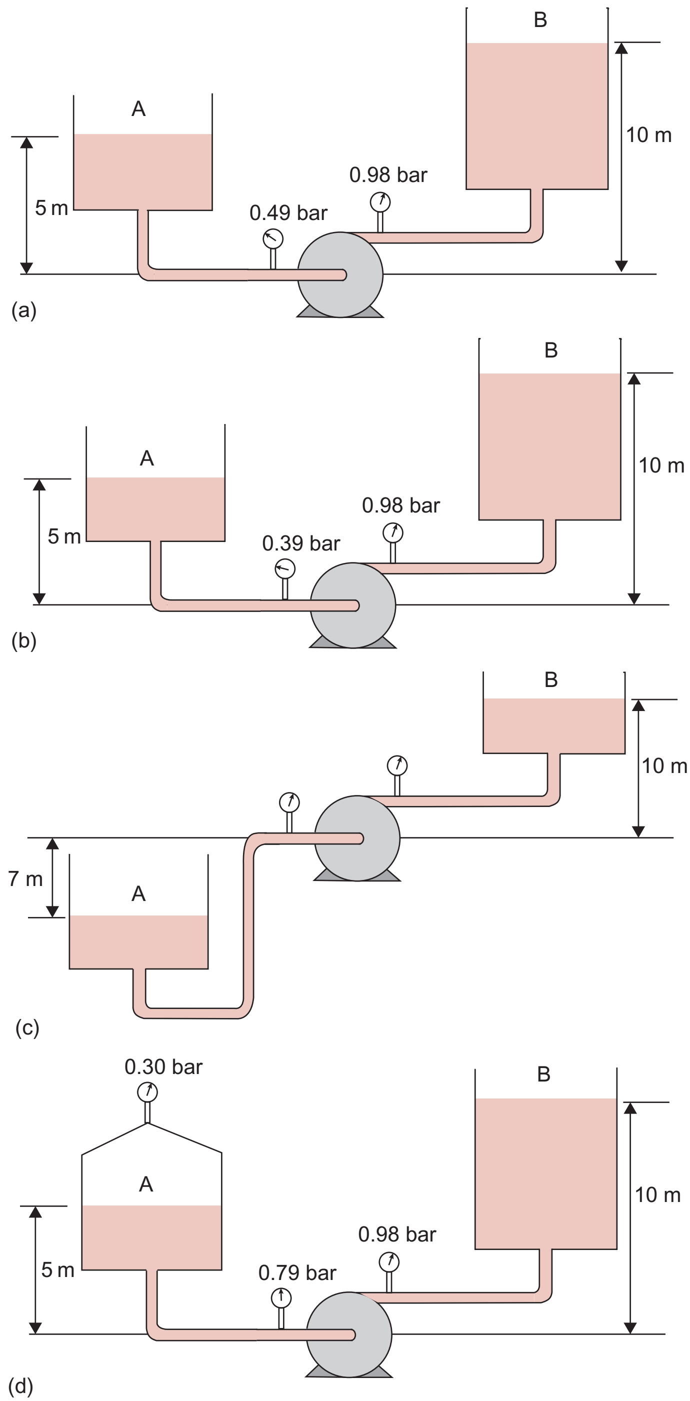


Figure 2.22 Suction and discharge pressures in a pumping system under different conditions.

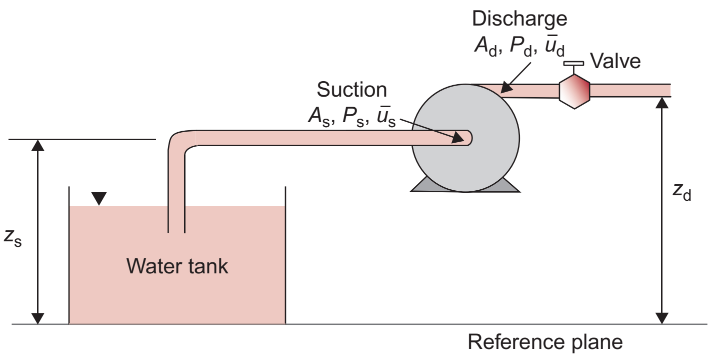


Figure 2.23 A test unit to determine performance of a pump.

the liquid flowing at a certain flow rate. In other words, we need to know the performance of a pump under certain operating conditions. This information, in the form of a pump characteristic diagram, is determined by the manufacturer and is supplied with the pump so that an engineer can make a proper judgment on the suitability of the pump for a given transport system.

Standard procedures have been developed to test the performance of industrial pumps (Hydraulic Institute, 1975). Testing of a pump is conducted by running the pump at a constant speed when set up on a test stand, as shown in Figure 2.23. The height of the suction port from a reference plane, z_s, and the height of the discharge port, z_d, are recorded. Initially, the valve is kept fully open and measurements of pressure on the suction and discharge sides, volumetric flow rates, and torque supplied to turn the pump shaft are measured. Then the discharge valve is slightly closed and the measurements are repeated. This procedure is continued until the valve is nearly closed. The valve is never completely closed, or else the pump may be damaged.

The pump performance test involves measurement of volumetric flow rate, $\dot{V}$, areas of suction and discharge ports, A_s and A_d, heights of the suction and discharge ports, pressures at suction, P_s and discharge, P_d. The data are then used in the following calculations.

The velocity at the suction is calculated as

$$\bar{u}_s = \frac{\dot{V}}{A_s} \tag{2.98}$$

Similarly, the velocity at the pump discharge is obtained as

$$\bar{u}_d = \frac{\dot{V}}{A_d} \tag{2.99}$$

The suction head, h_s, and discharge head, h_d, based on discussion presented in Section 2.5, are obtained as

$$h_s = \frac{\bar{u}_s^2}{2\alpha g} + z_s + \frac{P_s}{\rho g} \tag{2.100}$$

$$h_d = \frac{\bar{u}_d^2}{2\alpha g} + z_d + \frac{P_d}{\rho g} \tag{2.101}$$

The values of suction and discharge heads obtained from Equations (2.100) and (2.101) are used in calculating the **pump head** as

$$h_{pump} = h_d - h_s \tag{2.102}$$

Note that in Equation (2.102) we do not consider friction losses in pipes, since our interest at this point is primarily in the performance of the pump, not the system.

The power output of the pump is called the fluid power, Φ_{fl}. It is the product of the mass flow rate of the fluid and the pump head

$$\Phi_{fl} = \dot{m} g h_{pump} \tag{2.103}$$

The fluid power may also be expressed in terms of volumetric flow rate, $\dot{V}$, as,

$$\Phi_{fl} = \rho g \dot{V} h_{pump} \tag{2.104}$$

The power required to drive the pump is called the break power, Φ_{bk}. It is obtained from the torque supplied to the pump shaft, Ω, and the angular velocity of the shaft, ω,

$$\Phi_{bk} = \omega \Omega \tag{2.105}$$

The efficiency of the pump is calculated from these two values of power. It is the ratio between the power gained by the fluid and the power supplied by the shaft driving the pump, or,

$$\eta = \frac{\Phi_{fl}}{\Phi_{bk}} \tag{2.106}$$

The calculated values of pump head, efficiency, and break power are used to develop a pump characteristic diagram as discussed in the following section.

Example 2.13

The following data were collected while testing a centrifugal pump for water at 30°C. Suction pressure = 5 bar, discharge pressure = 8 bar, volumetric flow rate = 15,000 L/h. Calculate the pump head at the given flow rate and power requirements.

Given

Suction pressure $= 5$ *bar* $= 5 \times 10^5$ *Pa* $= 5 \times 10^5$ *N/m*$^2 = 5 \times 10^5$ *kg/(m s*^{2}*)*
Discharge pressure $= 8$ *bar* $= 8 \times 10^5$ *Pa* $= 8 \times 10^5$ *N/m*$^2 = 8 \times 10^5$ *kg/(m s*^{2}*)*
Volumetric flow rate $= 15{,}000$ *L/h* $= 0.0042$ *m*^{3}*/s*

Approach

We will use Equation (2.102) to obtain pump head and Equation (2.104) to calculate the fluid power requirements.

Solution

1. *In Equation (2.102), for the pump shown in Figure 2.23, the suction and discharge velocities are about the same and the difference in elevation* $z_2 - z_1$ *may be neglected, then*

$$h_{pump} = \frac{(P_d - P_s)}{\rho g}$$

$$h_{pump} = \frac{(8-5)\times 10^5\,[\mathrm{kg}/(\mathrm{m\,s}^2)]}{995.7\,[\mathrm{kg}/\mathrm{m}^3]\times 9.81\,[\mathrm{m}/\mathrm{s}^2]}$$

$$h_{pump} = 30.7\ \mathrm{m}$$

2. *From Equation (2.104)*

$$\Phi_{fl} = 995.7\,[\mathrm{kg}/\mathrm{m}^3]\times 9.81\,[\mathrm{m}/\mathrm{s}^2]\times 0.0042\,[\mathrm{m}^3/\mathrm{s}]\times 30.7\,[\mathrm{m}]$$
$$\Phi_{fl} = 1259\ \mathrm{W} = 1.26\ \mathrm{kW}$$

3. *The fluid power requirement is 1.26 kW at a flow rate of 15,000 L/h. The pump head is 30.7 m.*

2.6.4 Pump Characteristic Diagram

The calculated values of the pump head, efficiency, and break power plotted against volumetric flow rate (also called capacity) constitute the characteristic diagram of the pump, as shown in Figure 2.24. Typically, the pump characteristic diagrams are obtained for water. Therefore, if a pump is to be used for another type of liquid, the curves must be adjusted for the different properties of the liquid. As seen in Figure 2.24, a centrifugal pump can deliver flow rate from zero to maximum, depending upon the head and conditions at the suction. These curves depend upon the impeller diameter and the casing size. The relationship between the head and volumetric flow rate may be rising, drooping, steep, or flat. As seen in the figure, a rising head curve is shown, since the head increases with decreasing flow rate. The shape of the curve depends upon the impeller type and its design characteristics. At zero

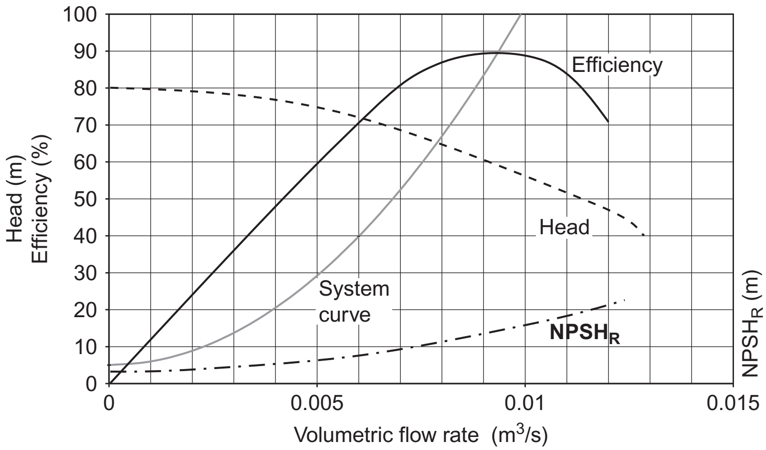


Figure 2.24 Performance characteristic curves for a pump.

capacity, when the discharge valve is completely shut, the efficiency is zero, and the power supplied to the pump is converted to heat.

We can draw several conclusions by examining the characteristic diagram of the pump. As the total head decreases, the volumetric flow rate increases. When the fluid level in the tank on the suction side decreases, the total head increases and the volumetric flow rate decreases. The efficiency of a pump is low both at low and high volumetric flow rates. The break power increases with the flow rate; however, it decreases as the maximum flow rate is reached.

The peak of efficiency curve represents the volumetric flow rate where the pump is most efficient. The flow rate at the peak efficiency is the design flow rate. The points on the head and power curve corresponding to the maximum efficiency are called best efficiency points, or BEP. With increasing volumetric flow rate, the power required to operate the pump increases. If a different impeller diameter is used, the head curve is shifted; increasing the diameter raises the curve. Thus, by using an impeller of a larger diameter pump, we can pump liquid to a higher head. Figure 2.24 also shows the net positive suction head (NPSH), which we will discuss in the following section.

2.6.5 Net Positive Suction Head

An important issue that requires our careful attention in designing pumps is to prevent conditions that may encourage vaporization of the liquid being transported. In a closed space, a certain pressure on the liquid surface is necessary to prevent vapors from escaping from the liquid. This pressure is the vapor pressure of the liquid. In a pumping system, it is important that the pressure of the liquid does not decrease below the vapor pressure of the liquid at that temperature. If it does so, a phenomenon called cavitation may occur at the eye of the impeller. As the liquid enters the eye of the impeller, pressure at this location is the lowest in the entire liquid handling system. If the pressure at that location is lower than the liquid vapor pressure, then vaporization of the liquid will begin.

Any formation of vapors will lower the efficiency of a pump. Furthermore, as the vapors travel further along the impeller toward the periphery, the pressure increases, and the vapors condense rapidly. Cavitation may be recognized as a crackling sound produced as vapor bubbles form and collapse on the impeller surface. With cavitation occurring at high frequency and extremely high local pressures, any brittle material such as the impeller surface may be damaged. To avoid cavitation, the pressure on the suction side must not be allowed to drop below the vapor pressure. The pump manufacturers specify the required net positive suction head ($NPSH_R$) as suction head minus the vapor pressure head, or:

$$NPSH_R = h_s - \frac{P_v}{\rho g} \tag{2.107}$$

where the total head on the suction side of a pump is

$$h_s = \frac{P_s}{\rho g} + \frac{u_s^2}{2g} \tag{2.108}$$

Then,

$$NPSH_R = \frac{P_s}{\rho g} + \frac{u_s^2}{2g} - \frac{P_v}{\rho g} \tag{2.109}$$

where P_v = vapor pressure of the liquid being pumped.

The $NPSH_R$ must be exceeded to ensure that cavitation will be prevented. The manufacturers test their pumps experimentally to determine the value of $NPSH_R$, and these values are provided graphically as shown in Figure 2.24. The pump user must ensure that the available net positive suction head ($NPSH_A$) for a given application is greater than the required $NPSH_R$ as specified by the manufacturer.

In using a pump for a given application, a calculation is first made to determine the $NPSH_A$, which depends on the given flow system. For example, in the case of the flow system shown in Figure 2.25 between locations (1) and (2), we may apply Equation (2.96) to obtain

$$\frac{P_{atm}}{\rho g} - z_1 = \frac{P_2}{\rho g} + \frac{\bar{u}_2^2}{2g} + h_{1-2} \tag{2.110}$$

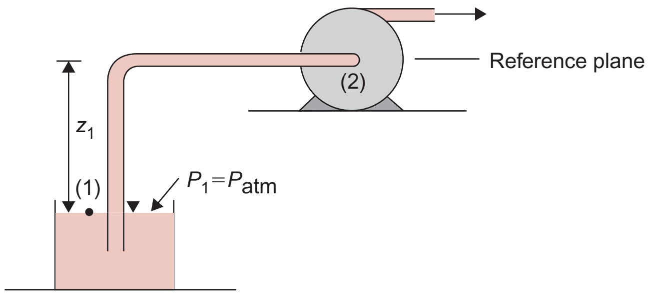


Figure 2.25 Suction side of a pumping system.

where h_{1-2} represents the major and minor losses between locations (1) and (2). The head available at the suction side of the pump (the pump impeller inlet) is

$$\frac{P_2}{\rho g} + \frac{\overline{u}_2^2}{2g} = \frac{P_{atm}}{\rho g} - z_1 - h_{1-2} \tag{2.111}$$

Then, the $NPSH_A$ is the suction head minus the vapor pressure head, or

$$NPSH_A = \frac{P_{atm}}{\rho g} - z_1 - h_{1-2} - \frac{P_v}{\rho g} \tag{2.112}$$

To avoid cavitation, an engineer must ensure that $NPSH_A$ is equal to or greater than $NPSH_R$. Note that in Equation (2.112) the $NPSH_A$ decreases if the height of the pump above the liquid surface in the reservoir, z_1, is increased, or if installing more fittings on the suction side increases the friction head loss, h_{1-2}.

Example 2.14

A centrifugal pump is to be located 4 m above the water level in a tank. The pump will operate at a rate of 0.02 m^3/s. The manufacturer suggests a pump with a $NPSH_R$ at this flow rate as 3 m. All frictional losses may be neglected except a heat exchanger between the pipe inlet and the pump suction that has a loss coefficient $C_f = 15$. The pipe diameter is 10 cm and the water temperature is 30°C. Is this pump suitable for the given conditions?

Given

Pipe diameter = 10 cm = 0.1 m
Pump location above water level in tank = 4 m
Volumetric flow rate = 0.02 m^3/s
Loss coefficient due to heat exchanger $C_f = 15$
Temperature of water = 30°C
$NPSH_R = 3$ *m*

Approach

We will first determine the head losses and then using Equation (2.112) determine the $NPSH_A$. From steam tables (Table A.4.2), we will determine vapor pressure at 30°C.

Solution

1. *Velocity is obtained from the volumetric flow rate using Equation (2.17):*

$$\bar{u} = \frac{0.02\,[m^3/s]}{\dfrac{\pi \times (0.1)^2 [m^2]}{4}}$$

$$\bar{u} = 2.55\ m/s$$

2. *The frictional head loss due to the heat exchanger is obtained using an equation similar to Equation (2.93):*

$$h_L = C_{heat\ exchanger} \frac{u^2}{2g}$$

$$h_L = \frac{15 \times (2.55)^2\,[m^2/s^2]}{2 \times 9.81\,[m/s^2]}$$

$$h_L = 4.97\ m$$

3. *From steam tables, at 30°C, vapor pressure is 4.246 kPa; then $NPSH_A$ is, from Equation (2.112),*

$$\frac{101.3 \times 1000\,[\mathrm{Pa}]}{9.81\,[m/s^2] \times 995.7\,[kg/m^3]} - 4\,[m] - 4.97\,[m] - \frac{4.246 \times 1000\,[Pa]}{9.81\,[m/s^2] \times 995.7\,[kg/m^3]}$$

(Note that 1 Pa = 1 kg/(m s^2))

$$NPSH_A = 10.37 - 4 - 4.97 - 0.43$$

$$NPSH_A = 0.97\ m$$

4. *The $NPSH_A$ is less than the $NPSH_R$. This suggests that cavitation will occur. Therefore the recommended pump is unsuitable for the given conditions. Another pump with an $NPSH_R$ of less than 0.97 m should be chosen to prevent cavitation.*

2.6.6 Selecting a Pump for a Liquid Transport System

In Section 2.6.3 we noted the two requirements in designing a liquid transport system—information about the pump and the system. So far we have examined the requirements of a pump. We will now consider a total liquid transport system containing pipes, valves, fittings, and other process equipment. Remember that the purpose of installing a pump in a liquid transport system is to increase the energy of the liquid so that it can be moved from one location to another. For example, in Figure 2.26, a pump is being used to pump liquid from tank A to tank B. The system contains a certain length of pipe, elbows, and a valve. The liquid level in tank A is z_1 from the ground. In tank B, the top of the liquid is z_2 from the ground. The velocity of the liquid surfaces at locations 1 and 2 is negligible, and in both tanks the surfaces are exposed to atmospheric pressure. Therefore, for this system,

$$h_{system} = z_2 - z_1 + h_{1-2} \tag{2.113}$$

From Equation (2.96), we observe that the friction losses, h_{1-2}, are proportional to the square of velocity. Since velocity is proportional to volumetric flow rate, the frictional losses are proportional to the square of the volumetric flow rate. Or,

$$h_{1-2} = C_{system}\dot{V}^2 \tag{2.114}$$

where C_{system} is a system constant. Thus, substituting Equation (2.114) in Equation (2.113),

$$h_{system} = z_2 = z_1 + C_{system}\dot{V}^2 \tag{2.115}$$

The system head as a function of the volumetric flow rate is shown in Figure 2.27a. The upward increasing curve is due to the quadratic function in Equation (2.115). The system head, h_{system}, depends upon the change in elevation (total static head) and any of the major and minor

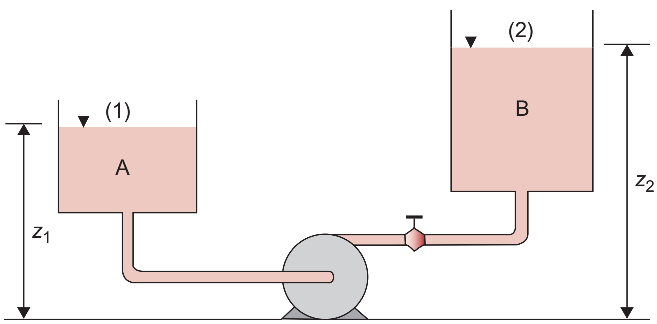
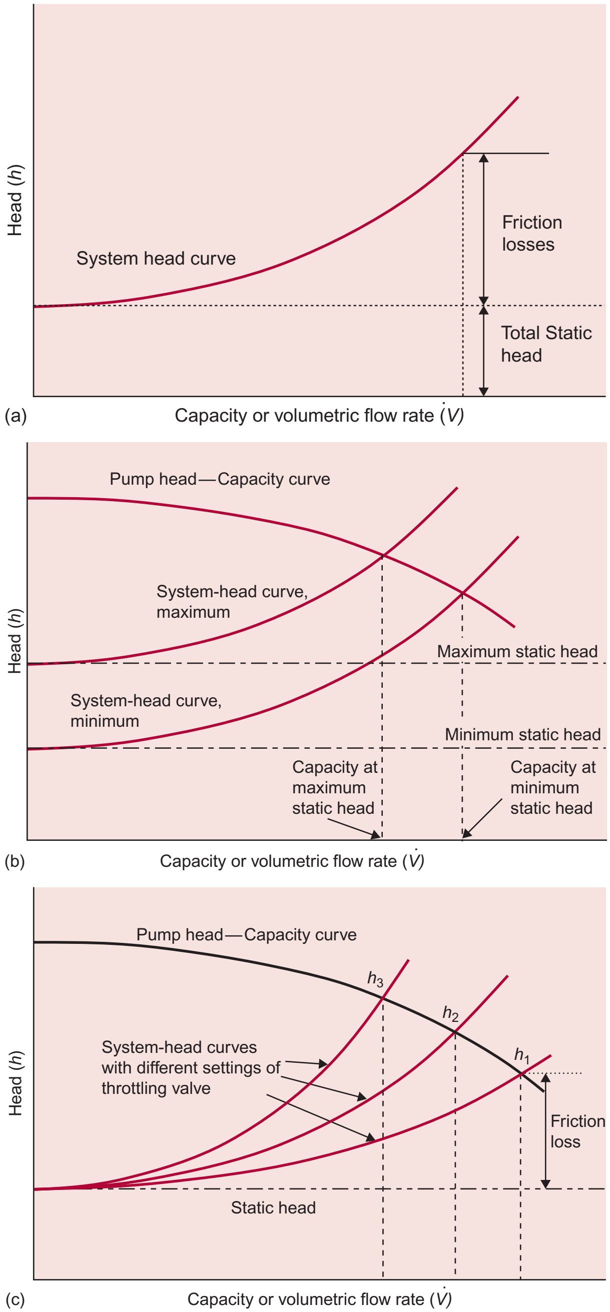


Figure 2.26 Pumping liquid from one tank to another.



■ **Figure 2.27** (a) System-head curve for a pump. (b) System-head curves for minimum and maximum static head. (c) System-head curves with different settings of throttling valve.

losses. In Figure 2.27b, two system head curves are shown, indicating situations where the static head may be changing. Similarly, if the frictional losses change, for example if a valve in a pipeline is closed, or the pipe becomes fouled over a period of time, then the friction loss curve may shift as shown in Figure 2.27c, indicating three different friction loss curves. Note that in Figures 2.27b and 2.27c, the pump-head curve obtained from the pump manufacturer, as discussed in Section 2.6.4, is also drawn. The intersection of the system-head curve and pump-head curve gives the operating point of the selected pump that is in agreement with the system requirements.

Therefore, to determine the operating conditions for a given liquid transport system, such as the one shown in Figure 2.26, the system curve is superimposed on the characteristic diagram of the pump, as shown in Figure 2.28. The intersection of the system curve and the pump performance curve, A, called the operating point, gives the operating values of flow rate and head. These two values satisfy both the system curve and the pump performance curve.

Typically, the operating point should be near the maximum value of the efficiency of the pump. However, this point depends upon the system curve. The curve will shift if there are increased losses, for example due to an increased number of fittings. Similarly, due to fouling of internal pipe surfaces, the frictional losses within the pipe may increase. If the system curve moves more toward the left, the new operating point, B, will be at a lower efficiency as seen in Figure 2.28.

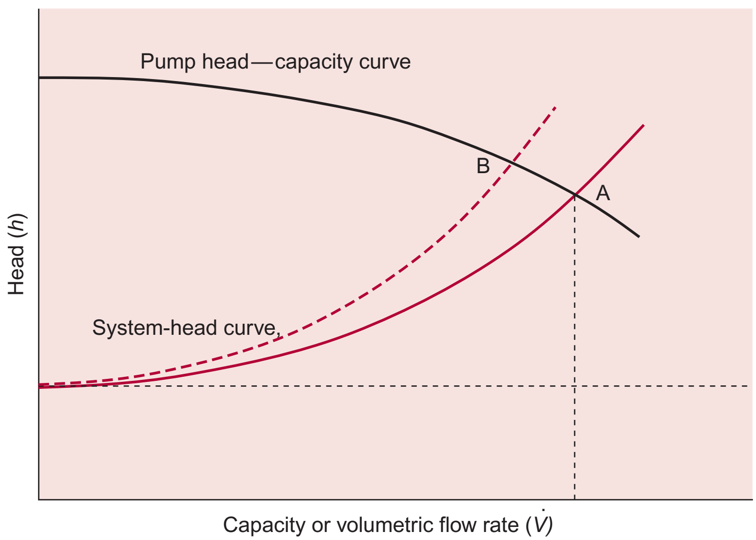


Figure 2.28 Pump head—capacity curve and system-head curve.

Example 2.15

A centrifugal pump is being considered for transporting water from tank A to tank B. The pipe diameter is 4 cm. The friction factor is 0.005. The minor losses include a contraction at the pipe inlet, expansion at the pipe outlet, four pipe bends, and a globe valve. The total length is 25 m and the elevation difference between the levels of water in tank A and B is 5 m. The performance characteristics of the pump supplied by the manufacturer are given in Figure E2.5.

Given

Pipe diameter = 4 cm = 0.04 m

Pipe length = 25 m

Friction factor = 0.005

C_f *(elbow) = 1.5 (from Table 2.2)*

C_f *(globe valve, fully opened) = 10 (from Table 2.2)*

$C_{fc} = 0.5$ *(from Equation (2.90) for* $D_1 \gg D_2$*)*

$C_{fe} = 1.0$ *(from Equation (2.92) for* $D_2 \gg D_1$*)*

Approach

We will apply the energy expression, Equation (2.96), between locations (1) and (2). Then we will express it in terms of pump head vs flow rate. We will plot it on the performance curve and determine the intersection point.

Solution

1. *Applying the energy Equation (2.96), noting that* $P_1 = P_2 = 0$, $\bar{u}_1 = \bar{u}_2$, $z_2 - z_1 = 5$ m, *we obtain*

$$h_{pump} = z_2 - z_1 + \text{major losses} + \text{minor losses}$$

$$h_{pump} = 5[m] + \left\{ \frac{4 \times 0.005 \times 25[m]}{0.04[m]} + 0.5 + 1.0 + 4(1.5) + 10 \right\} \times \frac{\bar{u}^2[m^2/s^2]}{2 \times 9.81[m/s^2]}$$

$$h_{pump} = 5 + 1.5291 \times \bar{u}^2$$

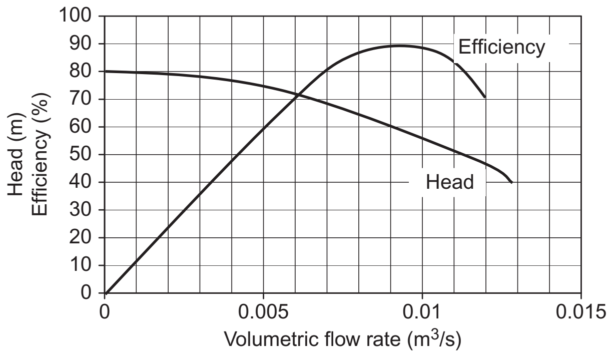
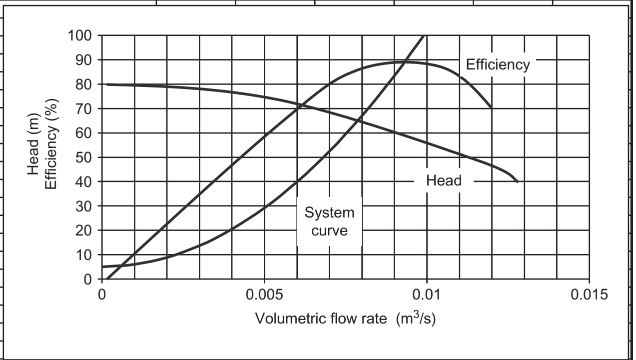


Figure E2.5 Performance characteristic of a centrifugal pump.

	A	B	C	D	E	F	G
1							
2	Volumetric flow rate	Pump head					
3	0	5	← =5+968283*C3^2				
4	0.001	6.0					
5	0.002	8.9					
6	0.003	13.7					
7	0.004	20.5					
8	0.005	29.2					
9	0.006	39.9					
10	0.007	52.4					
11	0.008	67.0					
12	0.009	83.4					
13	0.01	101.8					



■ **Figure E2.6** System curve plotted with pump performance curves for Example 2.15.

2. *Velocity may be expressed in terms of volumetric flow rate, using Equation (2.17) as*

$$\overline{u} = \frac{4\dot{V}[m^3/s]}{\pi(0.04)^2[m^2]}$$

3. *Substituting $\overline{u}$ in the expression for h_{pump} in Step (1),*

$$h_{pump} = 5 + 968{,}283 \times \dot{V}^2$$

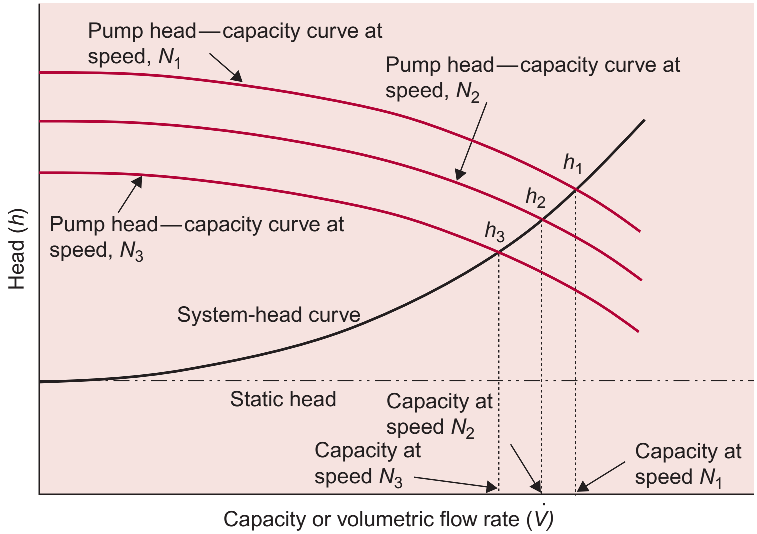
4. *Plotting the expression for h_{pump} obtained in Step (3) in Figure E2.6 we determine the operating point where the system curve intersects the head curve. The volumetric flow rate at the operating point is 0.0078 m^3/s with a head of 65 m, and an efficiency of 88%. This operating efficiency is close to the peak efficiency of 90%.*
5. *The pump head needed at the pump shaft:*

$$= \frac{65\ [m]}{0.88} = 73.9\ m$$

6. *The break power needed to drive the pump is obtained from Equations (2.104) and (2.106) as*

$$= \frac{990[kg/m^3] \times 9.81[m/s^2] \times 0.0078\ [m^3/s] \times 65\ [m]}{0.88}$$

$$= 5.6\ kW$$



■ **Figure 2.29** Pump head—capacity curve at different pump speeds.

2.6.7 Affinity Laws

A set of formulas known as affinity laws govern the performance of centrifugal pumps at various impeller speeds. These formulas are as follows:

$$\dot{V}_2 = \dot{V}_1(N_2/N_1) \tag{2.116}$$

$$h_2 = h_1(N_2/N_1)^2 \tag{2.117}$$

$$\Phi_2 = \Phi_1(N_2/N_1)^3 \tag{2.118}$$

where N is impeller speed, $\dot{V}$ is volumetric flow rate, Φ is power, and h is head.

These equations can be used to calculate the effects of changing impeller speed on the performance of a given centrifugal pump. For example, Figure 2.29 shows the pump head curve for three different impeller speeds that may be obtained with the use of a variable speed motor operating the pump. Example 2.16 illustrates the use of these formulas.

Example 2.16

A centrifugal pump is operating with the following conditions:
volumetric flow rate = 5 m^3/s
total head = 10 m
power = 2 kW
impeller speed = 1750 rpm
Calculate the performance of this pump if it is operated at 3500 rpm.

Solution

The ratio of speeds is

$$\frac{N_2}{N_1} = \frac{3500}{1750} = 2$$

Therefore, using Equations (2.116), (2.117), and (2.118),

$$\dot{V}_2 = 5 \times 2 = 10\ m^3/s$$
$$h_2 = 10 \times 2^2 = 40\ m$$
$$P'_2 = 2 \times 2^3 = 16\ kW$$

2.7 FLOW MEASUREMENT

The measurement of flow rate in a liquid transport system is an essential component of the operation. As illustrated in previous sections, knowledge of flow rate and/or fluid velocity is important in design calculations. In addition, periodic measurements during actual operations are required to ensure that system components are performing in an expected manner.

There are several types of flow measurement devices that are inexpensive and lead to direct quantification of the mass flow rate or velocity. These methods include (a) Pitot tube, (b) orifice meter, and (c) venturi tube. With all three of these methods, a portion of the measurement involves pressure difference. The device most often used for this purpose is a U-tube manometer. Let us first consider how a U-tube manometer is used in measuring pressure, then we will examine its use in flow measuring devices.

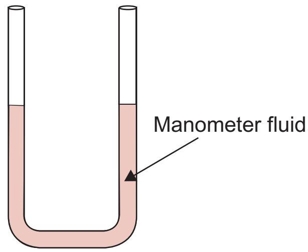


Figure 2.30 A manometer.

A U-tube manometer is a small-diameter tube of constant diameter shaped as a "U" as shown in Figure 2.30. The tube is partially filled with a fluid called the manometer fluid, rising to a certain height in each of its arms. This fluid must be different from the fluid whose pressure is to be measured. For example, mercury is a commonly used manometer fluid.

Let us consider a case where we want to measure pressure at location A in a vessel containing some fluid, as shown in Figure 2.31. For this purpose, a hole is drilled in the side of the vessel at the same elevation as location A, and one arm of the U-tube manometer is connected to this hole. As shown in the figure, the pressure of the fluid in the vessel pushes the manometer fluid down in the left-hand arm while raising it by an equal distance in the right-hand arm. After the initial displacement, the

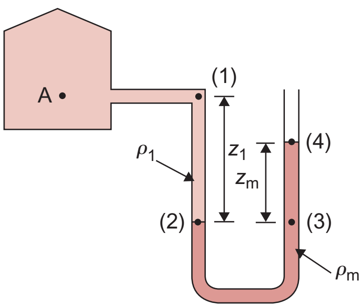


Figure 2.31 A manometer used to measure pressure in a chamber.

manometer fluid comes to rest. Therefore, we can apply the expressions developed in Section 1.9 for static head.

An easy approach to analyze the pressures at various locations within the U-tube manometer is to account for pressures at selected locations starting from one side of the manometer and continuing to the other. Using this approach, we note that the pressure at location (1) is same as at A because they are at the same elevation. From location (1) to (2), there is an increase in pressure equivalent to $\rho_1 g z_1$. Pressures at locations (2) and (3) are the same because they are at the same elevation and the fluid between locations (2) and (3) is the same. From location (3) to (4) there is a decrease in pressure equal to $\rho_m g z_m$. The manometer fluid at location (4) is exposed to atmosphere. Therefore, we may write an expression as follows

$$P_A + \rho_1 g z_1 + \rho_m g z_m = P_{atm} \tag{2.119}$$

or

$$P_A = \rho_m g z_m - \rho_1 g z_1 + P_{atm} \tag{2.120}$$

If the density of manometer fluid, ρ_m, is much larger than the fluid in the vessel, ρ_1, then pressure at location A in the vessel is simply

$$P_A = \rho_m g z_m + P_{atm} \tag{2.121}$$

Therefore, knowing the difference between the heights of the manometer fluid in the two arms, z_m, and the density of the manometer fluid, we can determine the pressure at any desired location in the vessel. Note that the lengths of the manometer arms have no influence on the measured pressure. Furthermore, the term $\rho_m g z_m$ in Equation (2.121) is the gauge pressure according to Section 1.9.

Next, let us consider a case where a U-tube manometer is connected to two vessels containing fluids of different densities ρ_A and ρ_B, and under different pressures (Fig. 2.32). Assume that pressure in vessel A is greater than that in vessel B. Again we will approach this problem by tracking pressures from one arm of the manometer to another. Pressure at location (1) is the same as at A. From (1) to (2) there is an increase in pressure equivalent to $\rho_A g z_1$. Pressures at locations (2) and (3) are the same since they are at the same elevation and contain the same fluid. From (3) to (4), there is a decrease in pressure equal to $\rho_m g z_m$. From location (4) to (5) there is another decrease in pressure equal to $\rho_B g z_3$. The pressures at location B and (5) are the same. Thus we may write an expression for pressure as follows:

$$P_A + \rho_A g z_1 - \rho_m g z_m - \rho_B g z_3 = P_B \tag{2.122}$$

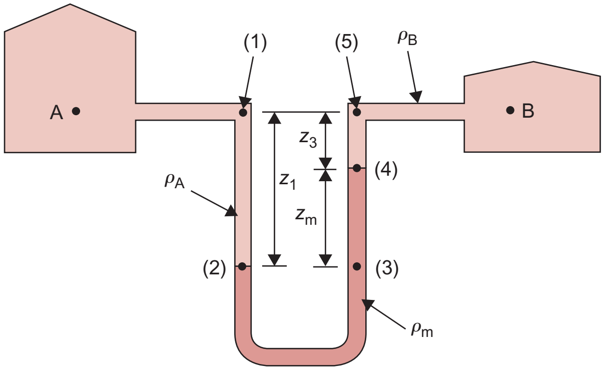


Figure 2.32 A manometer connected to two pressurized chambers.

or

$$P_A - P_B = g(\rho_m z_m - \rho_A z_1) + \rho_B g z_3 \tag{2.123}$$

As is evident from these derivations, the manometer fluid must have a higher density than the fluid whose pressure is being measured. Furthermore, the two fluids must be immiscible. The common manometer fluids are mercury and water, depending upon the application. An extension of the preceding analysis involves determining pressure difference in a flow-measuring device using a U-tube manometer. We will consider this analysis for different types of flow-measuring devices.

In pressure measurements involving fluid flow, it is important to note that there are typically three kinds of pressures involved: static, dynamic, and stagnation pressure.

Static pressure is the actual thermodynamic pressure of a moving fluid depicted by the first term in the Bernoulli equation (Equation (2.67)). As shown in Figure 2.33, the pressure of the moving fluid measured at location (1) is the static pressure. If the pressure sensor were moving with the fluid at the same velocity as the fluid, then the fluid would appear "static" to the sensor, hence the name. A common procedure to measure static pressure is to drill a hole in the duct, making sure that there are no imperfections in the hole such as burrs, so that the fluid moving in the duct is not disturbed. A pressure-measuring device such as a piezometer tube (a) is connected to the hole at location (2) to measure the static pressure as shown in Figure 2.33. Using the same approach as for a U-tube manometer to track pressures at various locations, as discussed in the previous section, the pressure at location (1) is

$$P_1 = P_3 + \rho g z_2 + \rho g z_1 \tag{2.124}$$

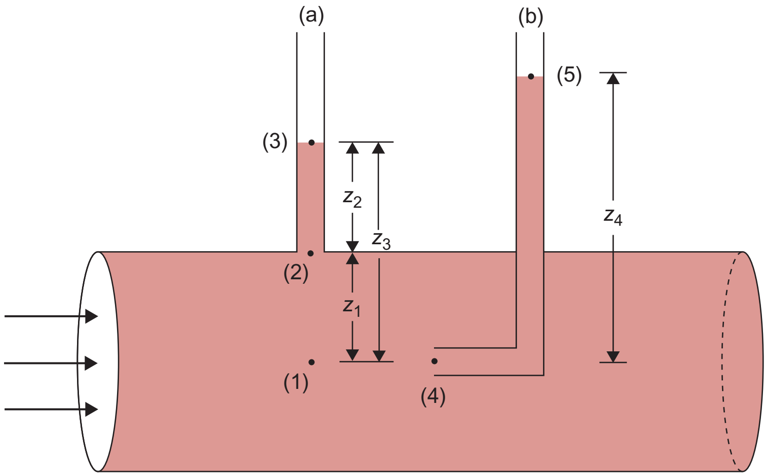


Figure 2.33 Measuring static and velocity head in fluid flow.

Since $P_3 = 0$, because it is the gauge atmospheric pressure, then

$$P_1 = \rho g(z_1 + z_2) = \rho g z_3 \tag{2.125}$$

If we insert a thin tube (b) into the duct, as shown in Figure 2.33, some of the fluid will be pushed through the tube to height z_4 in the tube. After a short transient period, the fluid inside the tube will come to rest and its velocity will be zero. This would imply that at the entrance of the tube, at location (4), the fluid velocity is zero and it is stagnant. Therefore, the pressure of the fluid at (4) will be the **stagnation pressure**. Applying the Bernoulli equation to locations (1) and (4), assuming they are at the same elevation, we have

$$\frac{P_1}{\rho} + \frac{u_1^2}{2g} = \frac{P_4}{\rho} + \cancel{\frac{u_4^2}{2g}}^{\,0} \tag{2.126}$$

Therefore, the stagnation pressure, P_4, is

$$P_4 = P_1 + \frac{\rho u_1^2}{2g} \tag{2.127}$$

In Equation (2.127), the term $\rho u_1^2/2g$ is called the **dynamic pressure** because it represents the pressure due to the fluid's kinetic energy. The stagnation pressure, P_4, is the sum of static and dynamic pressures, and it is the highest pressure obtainable along a given stream line assuming that elevation effects are negligible. Note the levels of fluid shown in the two tubes in Figure 2.33; the difference in the levels between tubes (a) and (b) is the kinetic energy term. We will use these definitions of pressure in developing a design equation for a Pitot tube sensor that is commonly used to measure fluid velocity.

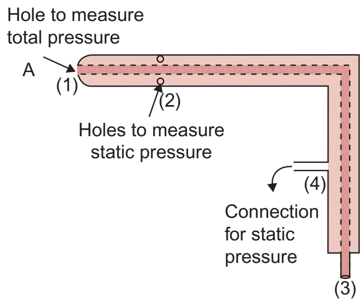


Figure 2.34 A pitot tube.

2.7.1 The Pitot Tube

A Pitot tube is a widely used sensor to measure velocity of a fluid. The design principle is based on the existence of stagnation and static pressures when an object is placed in the flowing fluid. A schematic of a Pitot[4] tube is presented in Figure 2.34. As indicated, the system is designed with two small concentric tubes, each leading to a separate outlet. The inlet hole of the inner tube is oriented directly into the fluid flow, whereas the inlet to the outer tube is through one or more holes located on the circumference of the outer tube. The outlets of the Pitot tube are connected to a U-tube manometer to measure the differential pressure. The inlet hole at location (1) measures the stagnation pressure. If the pressure and velocity in the fluid, at location A, upstream of (1), are P_A and u_A, and elevation difference between (1) and (3) is negligible, then

$$P_3 = P_A + \frac{\rho_f u_A^2}{2} \tag{2.128}$$

At location (2), the static pressure is measured. If the elevation difference between locations (2) and (4) is negligible then

$$P_4 = P_2 = P_A \tag{2.129}$$

Then, from Equations (2.128) and (2.129),

$$P_3 - P_4 = \frac{\rho_f u_A^2}{2} \tag{2.130}$$

or, rearranging,

$$u_A = \sqrt{\frac{2(P_3 - P_4)}{\rho_f}} \tag{2.131}$$

We obtained Equation (2.131) using the Bernoulli equation, which requires the fluid to be inviscid (viscosity = 0). This equation may be modified for real fluids by introducing a tube coefficient, C,

$$u_A = C\sqrt{\frac{2(P_3 - P_4)}{\rho_f}} \tag{2.132}$$

[4] Henri Pitot (1695–1771) was a French hydraulic engineer who began his career as a mathematician. In 1724, he was elected to the French Academy of Sciences. In Montpellier, he was in charge of constructing an aqueduct that included a 1-km-long Romanesque stone arch section. His studies included flow of water in rivers and canals, and he invented a device to measure fluid velocities.

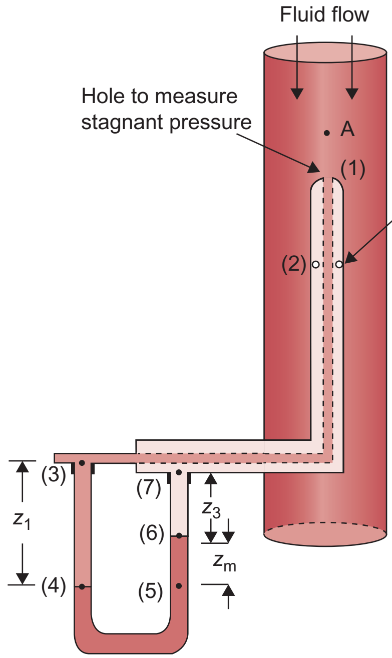


Figure 2.35 A pitot tube used to measure velocity of a fluid flowing in a pipe.

Equation (2.132) indicates that the fluid velocity in a stream at any desired location can be determined using a Pitot tube by measurement of pressure difference $P_3 - P_4$. The density ρ_f of the fluid and a tube coefficient C must be known. In most cases, $C \leq 1.0$. The velocity measured by the Pitot tube is the fluid velocity at the location A, upstream of the tip of the Pitot tube. To obtain an average velocity in a duct, several measurements are necessary.

If a U-tube manometer is used with the Pitot tube as shown in Figure 2.35, then we can follow the same approach as given earlier in this section to account for the pressures at various locations. Therefore, in Figure 2.35, from location (3) to (4), there will be an increase in pressure equal to $\rho_f g z_1$. Pressure at locations (4) and (5) will be the same as they are at the same elevation. There will be a decrease in pressure from (5) to (6) equal to $\rho_m g z_m$. From locations (6) to (7) there is additional decrease in pressure equal to $\rho_f g z_3$. Thus we may write the following:

$$P_3 + \rho_f g z_1 - \rho_m g z_m - \rho_f g z_3 = P_7 \quad (2.133)$$

or

$$P_3 + \rho_f g(z_1 - z_3) - \rho_m g z_m = P_7 \quad (2.134)$$

or rearranging terms,

$$P_3 - P_7 = g z_m(\rho_m - \rho_f) \tag{2.135}$$

By introducing Equation (2.135) into Equation (2.132), and noting that $P_3 - P_4$ in Figure 2.34 is analogous to $P_3 - P_7$ in Figure 2.35, we obtain

$$u_A = C\sqrt{\frac{2g(\rho_m - \rho_f)z_m}{\rho_f}} \tag{2.136}$$

We can measure the velocity directly from the change in height of a manometer fluid (z_m) when the two sides of the U-tube manometer are connected to the two outlets from the Pitot tube. The only other requirements for Equation (2.136) are knowledge of the fluid densities ρ_m and ρ_f, acceleration due to gravity (g), and the tube coefficient C.

Example 2.17

A Pitot tube is being used to measure maximum velocity for water flow in a pipe. The tube is positioned with the inlet to the inner tube along the center axis of the pipe. A U-tube manometer gives a reading of 20 mm Hg. Calculate the velocity of water, assuming a tube coefficient of 1.0. The density of mercury is 13,600 kg/m^3.

Given

Manometer reading = 20 mm Hg = 0.02 m Hg
Density (ρ_m) of mercury = 13,600 kg/m^3
Density (ρ) of water = 998 kg/m^3
Tube coefficient (C) = 1.0

Approach

By using Equation (2.136), the velocity of water can be computed.

Solution

1. *Using Equation (2.136) with C = 1,*

$$\bar{u}_2 = 1.0\left[\frac{2(9.81\ m/s^2)}{998\ kg/m^3}(13{,}600\ kg/m^3 - 998\ kg/m^3)(0.02\ m)\right]^{1/2}$$

$$\bar{u}_2 = 2.226\ m/s$$

2.7.2 The Orifice Meter

By introducing a restriction of known dimensions into flow within a pipe or tube, we can use the relationship between pressure across the

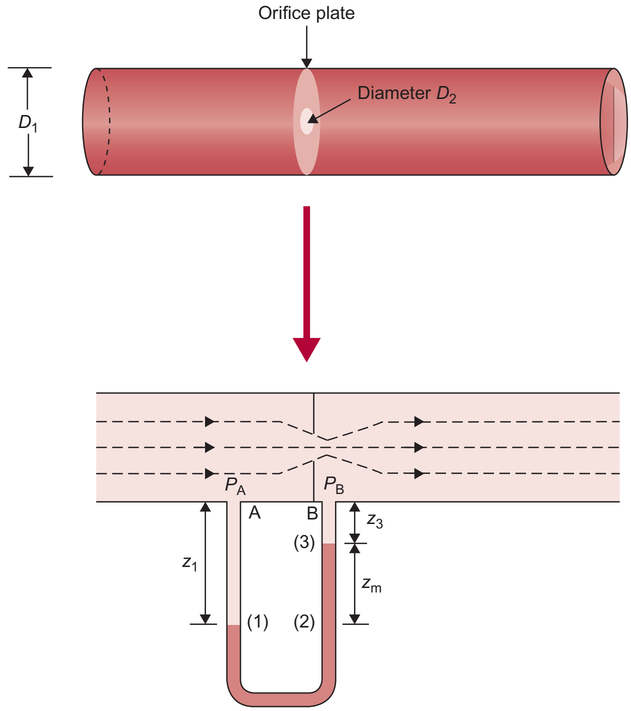


Figure 2.36 An orifice plate used to measure fluid flow.

restriction and velocity through the restriction to measure fluid flow rate. An orifice meter is a ring introduced into a pipe or tube that reduces the cross-sectional area of tube by a known amount. By attaching pressure taps or transducers at locations upstream and downstream from the orifice, the pressure changes can be measured.

We again use Equation (2.66) to analyze flow characteristics in the region near the orifice. Reference location A should be at sufficient distance upstream that the orifice does not influence flow characteristics. Reference location B is just slightly downstream from the orifice, where velocity is the same as within the orifice. Figure 2.36 illustrates the flow stream profile around the orifice meter and the reference locations. The pipe diameter is D_1 and the orifice diameter is D_2. Using Equation (2.66)

$$\frac{\overline{u}_A^2}{2} + \frac{P_A}{\rho_f} = \frac{\overline{u}_B^2}{2} + \frac{P_B}{\rho_f} \tag{2.137}$$

and

$$\overline{u}_A = \frac{A_2}{A_1}\overline{u}_B = \frac{D_2^2}{D_1^2}\overline{u}_B \tag{2.138}$$

By combining Equations (2.137) and (2.138),

$$\frac{\overline{u}_B^2}{2} + \frac{P_B}{\rho_f} = \left(\frac{D_2}{D_1}\right)^4 \frac{\overline{u}_B^2}{2} + \frac{P_A}{\rho_f} \tag{2.139}$$

or

$$\overline{u}_B = C\left\{\frac{2(P_A - P_B)}{\rho_f\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}\right\}^{1/2} \tag{2.140}$$

which allows computation of the velocity at location A from the pressure difference $P_A - P_B$ and the diameter of the pipe or tube D_1 and the orifice diameter D_2. Note that C is the orifice coefficient.

If we use a U-tube manometer to determine pressure drop, then we can use the same approach as shown earlier in Section 2.7.1 as follows. From Figure 2.36 we account for pressures. Moving from location A to (1), there is a pressure increase of $\rho_f g z_1$. Pressures at locations (1) and (2) are the same. From location (2) to (3), there is a pressure decrease equal to $\rho_m g z_m$. From location (3) to B there is a pressure decrease of $\rho_f g z_3$. Thus, we may write

$$P_A + \rho_f g z_1 - \rho_m g z_m - \rho_f g z_3 = P_B \tag{2.141}$$

Rearranging terms,

$$P_A - P_B = \rho_f g(z_3 - z_1) + \rho_m g z_m \tag{2.142}$$

or

$$P_A - P_B = z_m g(\rho_m - \rho_f) \tag{2.143}$$

By introducing Equation (2.140), the following relationship is obtained:

$$\overline{u}_B = C\left\{\frac{2g\left(\frac{\rho_m}{\rho_f} - 1\right) z_m}{\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}\right\}^{1/2} \tag{2.144}$$

which allows computation of average velocity in the fluid stream from the change in manometer fluid height and the density of the manometer fluid.

The magnitude of the orifice coefficient C is a function of exact location of the pressure taps, the Reynolds number, and the ratio of pipe diameter to orifice diameter. At $N_{Re} = 30{,}000$, the coefficient C will have a value of 0.61, and the magnitude will vary with N_{Re} at lower values. It is recommended that orifice meters be calibrated in known flow conditions to establish the exact values of the orifice coefficient.

Example 2.18

An orifice meter is being designed to measure steam flow in a food processing plant. The steam has a mass flow rate of approximately 0.1 kg/s in a 7.5-cm diameter (ID) pipe with a pressure of 198.53 kPa. Determine the density of the manometer fluid to be used so that pressure differences can be detected accurately and reasonably. A manometer of less than 1 m in height can be considered reasonable.

Given

Mass flow rate ($\dot{m}$) *of steam* $= 0.1$ *kg/s*
Pipe diameter (D_1) $= 7.5$ *cm* $= 0.075$ *m*
Steam density (ρ) $= 1.12$ *kg/m*^{3} *from Table A.4.2 at pressure of 198.53 kPa*
Orifice coefficient (C) $= 0.61$ *at* $N_{Re} = 30{,}000$

Approach

To use Equation (2.144) to compute density of manometer fluid (ρ_m), *the orifice diameter* D_2 *and manometer fluid height* z_m *must be assumed.*

Solution

1. *By assuming an orifice diameter* D_2 *of 6 cm or 0.06 m,*

$$\bar{u} = \frac{\dot{m}}{\rho A} = \frac{(0.1\ \text{kg/s})}{(1.12\ \text{kg/m}^3)[\pi(0.06\ \text{m})^2/4]} = 31.578\ \text{m/s}$$

2. *Since the manometer fluid height* (z_m) *must be less than 1 m, a value of 0.1 m will be assumed. Using Equation (2.144),*

$$31.578\ \text{m/s} = 0.61\left[\frac{2(9.81\ \text{m/s}^2)\left(\dfrac{\rho_m}{1.12\ \text{kg/m}^3} - 1\right)(0.1\ \text{m})}{1-(0.06/0.075)^4}\right]^{1/2}$$

$$\rho_m = 904.3\ \text{kg/m}^3$$

3. *This density could be obtained by using light oil with a density of 850 kg/m*^{3}.

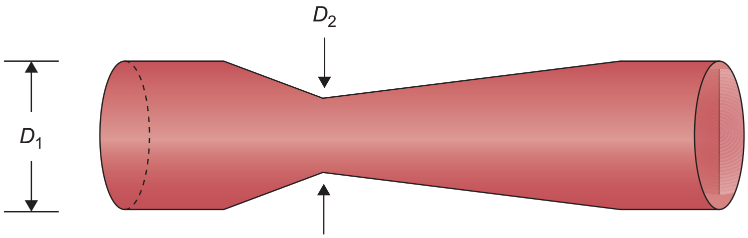


Figure 2.37 A venturi tube flow meter.

2.7.3 The Venturi Meter

To reduce energy loss due to friction created by the sudden contraction in flow in an orifice meter, a venturi tube of the type illustrated in Figure 2.37 can be used. An analysis similar to that presented for the orifice meter leads to the following equation:

$$\bar{u}_2 = C\left\{\frac{2g\left(\frac{\rho_m}{\rho_f}-1\right)z_m}{\left[1-\left(\frac{D_2}{D_1}\right)^4\right]}\right\}^{1/2} \tag{2.145}$$

where the average velocity $\bar{u}_2$ is at reference location 2, where diameter D_2 is the smallest value for the venturi. The venturi meter requires careful construction to ensure proper angles of entrance to and exit from the venturi. The meter requires a significant length of pipe for installation compared with the orifice meter. In general, the orifice meter is less costly and simpler to design than the venturi meter.

2.7.4 Variable-Area Meters

The flow meters considered in the preceding sections, namely the orifice meter and venturi tube, involve a change in flow rate through a constant cross-sectional area that generates a variable pressure drop as a function of flow rate. In a variable-area meter, the fluid stream is throttled by a constriction arranged in a manner such that the cross-sectional area is varied. This allows a variation of flow while maintaining a nearly constant pressure drop. The cross-sectional area in these devices is related to the flow rate by proper calibration.

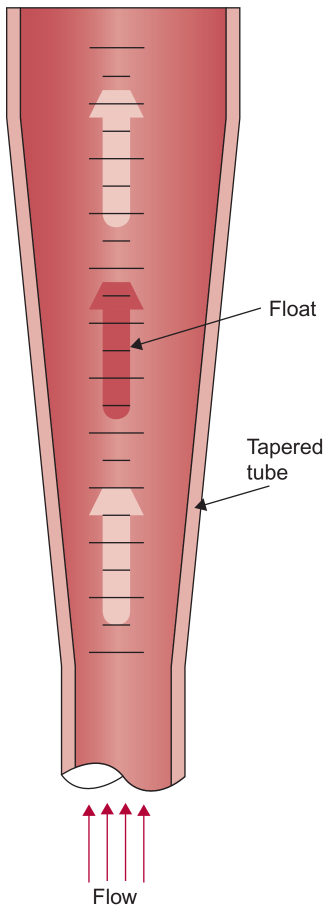


Figure 2.38 A variable flow meter.

A popular type of variable flow meter is a **rotameter**, shown in Figure 2.38. In this device, the height of a plummet, also called bob or float, in a tapered tube indicates the flow. The float moves up or down in a vertically mounted tapered tube, with the largest diameter of the tube at the top. The fluid moves up from the bottom and lifts the float. Because of the higher density of the float, the passage remains blocked until the pressure builds up and the buoyant effect of the fluid lifts the float; the fluid then flows between the float and the tube wall. As the passage for flow increases, a dynamic equilibrium is established between the position of the float and the pressure difference across the float and the buoyant forces. A scale mounted on the outside of the tube provides a measurement of the vertical displacement of the float from a reference point. The fluid flow in the tube can thus be measured. As the float is raised higher in the tapered tube, a greater area becomes available for the fluid to pass through; this is why this meter is called a *variable-area flow meter*.

The tube material is typically glass, acrylic, or metal. For measuring low flow rates, a ball or plumb-bob float shape is used, whereas for high flow capacities and applications that require high accuracy with constant viscosity, a streamlined float shape is used. The float materials commonly used include black glass, red sapphire, stainless steel, and tungsten. The capacities of rotameters are usually given in terms of two standard fluids: water at 20°C and air at 20°C at 101.3 kPa. A proper flow meter can be selected based on the float selection curves and capacity tables supplied by the manufacturers. A single instrument can cover a wide range of flow, up to tenfold; using floats of different densities, ranges up to 200-fold are possible. Unlike orifice meters, the rotameter is not sensitive to velocity distribution in the approaching stream. The installation of rotameters requires no straight section of pipe either upstream or downstream.

Industrial rotameters offer excellent repeatability over a wide range of flows. Their standard accuracy is $\pm 2\%$ of full scale, with capacities of 6×10^{-8} to 1×10^{-2} m^3/s of water and 5×10^{-7} to 0.3 m^3/s of air at standard temperature and pressure. Rotameters are also available for special requirements such as low volume, and high pressure. These instruments are calibrated when obtained from the manufacturer with a given size and shape bob and for a given bob density for a fluid of specified specific gravity.

2.7.5 Other Measurement Methods

In addition to methods in which pressure drop caused by a restriction in flow is measured, several methods have been developed for unique applications in the food industry. These methods vary considerably in operation principles but meet the needs for sanitary design.

Volumetric displacement as a flow measurement principle involves the use of a measuring chamber of known volume and containing a rotating motor. As flow is directed through the chamber, the rotor turns and displaces known volume magnitudes. The flow rate is detected by monitoring the number of revolutions of the rotor and accounting for the volume in each revolution.

Several flow measurement methods use ultrasound as a flow-sensing mechanism. Generally, these methods use the response from a high-frequency wave directed at the flow as an indication of flow rate. As the flow changes, the frequency changes. One method of flow detection uses the Doppler shift; changes in flow rate cause shifts in the wave frequency as the wave passes through the flow.

Another method of flow measurement is the use of a vortex created by inserting an object of irregular shape into the flow stream. Since the vortices move downstream with a frequency that is a function of flow rate, this frequency can be used as an indicator of flow rate. Typically, the frequencies are measured by placing heated thermistors in the vortex stream, followed by detection of cooling rates.

The flow rate of a fluid in a tube can be measured by placing a turbine wheel into the flow stream. As flow rate changes, the rotation speed changes in some proportional manner. Measurement of rotation is accomplished by using small magnets attached to the rotating part of the turbine. The magnets generate a pulse to be detected by a coil circuit located on the outside tube wall.

Each of the flow measurement methods described have unique features, and their use should be determined by the circumstances of the application. All have been used in various situations in the food industry.

2.8 MEASUREMENT OF VISCOSITY

Viscosity of a liquid can be measured using a variety of approaches and methods. The capillary tube and the rotational viscometer are the more common types of instruments used.

2.8.1 Capillary Tube Viscometer

Capillary tube measurement is based on the scheme shown in Figure 2.39. As shown, pressure (ΔP) is sufficient to overcome the shear forces within the liquid and produce flow of a given rate. The shear forces are operating on all internal liquid surfaces for the entire length L of tube and distance r from the tube center.

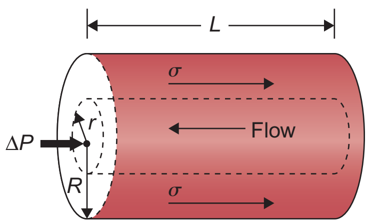
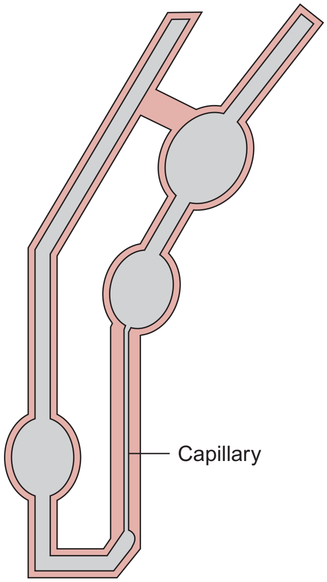


Figure 2.39 Force balance for a section of capillary tube.



■ Figure 2.40 A Cannon-Fenske viscometer.

Equation (2.39) provides the basis for design and operation of any capillary tube viscometer. For a tube with length L and radius R, measurement of a volumetric flow rate $\dot{V}$ at a pressure ΔP will allow determination of viscosity μ:

$$\mu = \frac{\pi \Delta P R^4}{8L\dot{V}} \tag{2.146}$$

Since Equation (2.146) is derived for a Newtonian fluid, any combination of flow rate and pressure drop will give the same viscosity value.

In a Cannon–Fenske type capillary viscometer, shown in Figure 2.40, we allow gravitational force to provide the pressure for liquid flow through the glass capillary tube. We can use a simple variation of the mathematical formulation developed for the capillary tube viscometer. By recognizing that

$$\Delta P = \frac{\rho V g}{A} \tag{2.147}$$

and the volumetric flow rate through the capillary tube is

$$\dot{V} = \frac{\text{volume of bulb}}{\text{discharge time}} = \frac{V}{t} \tag{2.148}$$

then, Equation (2.146) becomes

$$\mu = \frac{\pi \rho g R^4 t}{8V} \tag{2.149}$$

Equation (2.149) illustrates that viscosity of a liquid measured by a glass capillary tube will be a function of the liquid volume in the bulb, fluid density, the acceleration due to gravity ($g = 9.8$ m/s^2), and tube length L. We can determine viscosity by measuring the length of time for the liquid to drain from the bulb.

Example 2.19

A capillary tube viscometer is being used to measure the viscosity of honey at 30°C. The tube radius is 2.5 cm and the length is 25 cm. The following data have been collected:

ΔP (Pa)	$\dot{V}$ (cm^3/s)
10.0	1.25
12.5	1.55
15.0	1.80
17.5	2.05
20.0	2.55

Determine the viscosity of honey from the data collected.

Given

Data required to compute viscosity values from Equation (2.146), for example,

$\Delta P = 12.5$ Pa

$R = 2.5$ cm $= 0.025$ m

$L = 25$ cm $= 0.25$ m

$\dot{V} = 1.55\ \text{cm}^3/\text{s} = 1.55 \times 10^{-6}\ \text{m}^3/\text{s}$

Approach

The viscosity for each pressure difference (ΔP) and flow rate ($\dot{V}$) combination can be computed from Equation (2.146).

Solution

1. *Using Equation (2.146), a viscosity value can be computed for each $\Delta P - \dot{V}$ combination; for example,*

$$\mu = \frac{\pi(12.5\ \text{Pa})(0.025\ \text{m})^4}{8(0.25\ \text{m})(1.55 \times 10^{-6}\ \text{m}^3/\text{s})} = 4.948\ \text{Pa s}$$

2. *By repeating the same calculation at each $\Delta P - \dot{V}$ combination, the following information is obtained:*

ΔP (Pa)	$\dot{V}$ ($\times 10^{-6}$ m^3/s)	μ (Pa s)
10	1.25	4.909
12.5	1.55	4.948
15	1.8	5.113
17.5	2.05	5.238
20	2.55	4.812

3. *Although there is some variability with pressure (ΔP), there is no indication of a consistent trend, and the best estimate of the viscosity would be the arithmetic mean*

$$\mu = 5.004\ \text{Pa s}$$

2.8.2 Rotational Viscometer

The second type of viscometer is the rotational viscometer illustrated in Figure 2.41. This illustration is more specific for a coaxial-cylinder viscometer with the liquid placed in the space between the inner and outer cylinders. The measurement involves recording of torque Ω required to turn the inner cylinder at a given number of revolutions per unit time. To calculate viscosity from the measurements, the

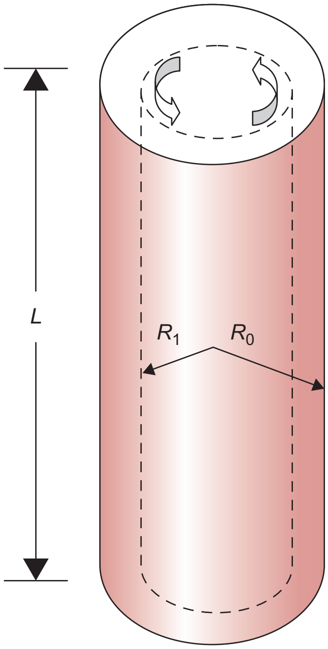


Figure 2.41 A coaxial-cylinder rotational viscometer.

relationships between torque Ω and shear stress σ, as well as revolutions per unit second N_r and shear rate $z_m\dot{\gamma}$ must be established.

The relationship between torque Ω and shear stress σ can be shown as

$$\Omega = 2\pi r^2 L\sigma \tag{2.150}$$

where the length L of the cylinder and the radial location r between the inner and outer cylinder are accounted for.

The angular velocity at r is

$$u = r\omega \tag{2.151}$$

Using differential calculus,

$$\frac{du}{dr} = \omega + \frac{r d\omega}{dr} \tag{2.152}$$

We note that ω does not contribute to shear. And the shear rate $\dot{\gamma}$ for a rotational system becomes a function of angular velocity ω, as follows:

$$\dot{\gamma} = -\frac{du}{dr} = r\left(-\frac{d\omega}{dr}\right) \tag{2.153}$$

By substitution of these relationships into Equation (2.150),

$$\frac{\Omega}{2\pi L r^2} = -\mu\left(r\frac{d\omega}{dr}\right) \tag{2.154}$$

To obtain the desired relationship for viscosity, an integration between the outer and inner cylinders must be performed:

$$\int_{o}^{\omega_i} d\omega = -\frac{\Omega}{2\pi\mu L}\int_{R_o}^{R_i} r^{-3} dr \tag{2.155}$$

where the outer cylinder (R_o) is stationary ($\omega = 0$) and the inner cylinder (R_i) has an angular velocity $\omega = \omega_i$. The integration leads to

$$\omega_i = \frac{\Omega}{4\pi\mu L}\left(\frac{1}{R_i^2} - \frac{1}{R_o^2}\right) \tag{2.156}$$

and since

$$\omega_i = 2\pi N_r \tag{2.157}$$

Note that ω is in units of radian/s and N_r is revolution/s. Then

$$\mu = \frac{\Omega}{8\pi^2 N_r L}\left(\frac{1}{R_i^2} - \frac{1}{R_o^2}\right) \tag{2.158}$$

Equation (2.158) illustrates that liquid viscosity can be determined using a coaxial-cylinder viscometer with an inner cylinder radius R_i, length L, and outer cylinder radius R_o by measurement of torque Ω at a given N_r (revolutions per second).

A variation of the coaxial-cylinder viscometer is the single-cylinder viscometer. In this device, a single cylinder of radius R_i is immersed in a container with the test sample. Then the outer cylinder radius R_o approaches infinity, and Equation (2.158) becomes

$$\mu = \frac{\Omega}{8\pi^2 N_r L R_i^2} \tag{2.159}$$

Several rotational viscometers operate using the single-cylinder principle, which assumes that the wall of the vessel containing the liquid during measurement has no influence on the shear stresses within the liquid. This may be a relatively good assumption for Newtonian liquids but should be evaluated carefully for each liquid to be measured.

Example 2.20

A single-cylinder rotational viscometer with a 1-cm radius and 6-cm length is being used to measure liquid viscosity. The following torque readings were obtained at several values of revolutions per minute (rpm):

N_r (rpm)	Ω ($\times 10^{-3}$ N cm)
3	1.2
6	2.3
9	3.7
12	5.0

Compute the viscosity of the liquid based on the information provided.

Given

Equation (2.159) requires the following input data (for example):

$\Omega = 2.3 \times 10^{-3}$ N cm $= 2.3 \times 10^{-5}$ N m

$N_r = 6$ rpm $= 0.1$ rev/s

$L = 6$ cm $= 0.06$ m

$R_i = 1$ cm $= 0.01$ m

Approach

Use Equation (2.159) to calculate viscosity from each rpm—torque reading combination.

Solution

1. *Using Equation (2.159) and the given data,*

$$\mu = \frac{(2.3 \times 10^{-5} \text{ N m})}{8\pi^2(0.1 \text{ rev/s})(0.06 \text{ m})(0.01 \text{ m})^2} = 0.485 \text{ Pa s}$$

2. *Using the same approach, values of viscosity are obtained for each N_r–Ω combination.*

N_r (rev/s)	Ω ($\times 10^{-5}$ N m)	μ (Pa s)
0.05	1.2	0.507
0.1	2.3	0.485
0.15	3.7	0.521
0.2	5.0	0.528

3. *Since it is assumed that the liquid is Newtonian, the four values can be used to compute an arithmetic mean of*

$$\mu = 0.510 \text{ Pa s}$$

2.8.3 Influence of Temperature on Viscosity

The magnitude of the viscosity coefficient for a liquid is influenced significantly by temperature. Since temperature is changed dramatically during many processing operations, it is important to obtain appropriate viscosity values for liquids over the range of temperatures encountered during processing. This temperature dependence of viscosity also requires that during measurements of viscosity, extra care must be taken to avoid temperature fluctuations. In the case of water, the temperature sensitivity of viscosity from Table A.4.1 is estimated to be 3%/°C at room temperature. This means that ±1% accuracy in its measurement requires the sample temperature to be maintained within ±0.3°C.

There is considerable evidence that the influence of temperature on viscosity for a liquid food may be described by an Arrhenius-type relationship:

$$\ln \mu = \ln B_A + \frac{E_a}{R_g T_A} \quad (2.160)$$

where B_A is the Arrhenius constant, E_a is an activation energy constant, and R_g is the gas constant. Equation (2.160) can be used to reduce the number of measurements required to describe the influence of temperature on viscosity of a liquid food. If we obtain values at three or more temperatures within the desired range and establish the magnitudes of the constants (B_A and E_a/R_g), we can predict with reasonable accuracy the viscosity coefficient at other temperatures within the range.

Example 2.21

For concentrated orange juice, Vitali and Rao (1984) obtained viscosity at a shear rate of 100 s^{-1} at eight different temperatures as shown in the following table. Determine the activation energy and the preexponential factor. Calculate the viscosity at 5°C.

Given

The values of viscosity at a shear rate of 100 s^{-1} at eight different temperatures are given in the table.

Temperature	Viscosity
−18.8	8.37
−14.5	5.32
−9.9	3.38
−5.4	2.22
0.8	1.56
9.5	0.77
19.4	0.46
29.2	0.28

Approach

We will use a spreadsheet first to convert the temperature to 1/T, where T is in absolute, K. Then we will plot ln (viscosity) versus 1/T. Using the trend line we will calculate the slope of the line, intercept, and regression coefficient. We will use the coefficients in the Arrhenius equation to obtain the viscosity at 5°C.

Solution

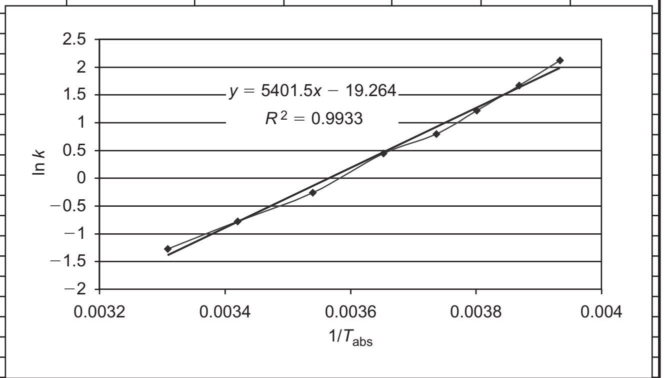
1. *Prepare a spreadsheet as shown in Figure E2.7.*
2. *Using the trendline feature of the Excel software, obtain the slope and intercept.*
3. *From the slope, $5401.5 \times R_g = 10{,}733.7$ cal/mol. Note that Gas Constant, $R_g = 1.98717$ cal/(mol K),*

   intercept, $B_A = 4.3 \times 10^{-9}$,
   and the regression coefficient = 0.99.

   The high regression coefficient indicates a high degree of fit.

Figure E2.7 Spreadsheet solution for Example 2.21.

	A	B	C	D	E	F	G
1	Temperature	1/Tabs	Viscosity	ln (viscosity)			
2	−18.8	0.00393391	8.37	2.12465388			
3	−14.5	0.003868472	5.32	1.6714733			
4	−9.9	0.003800836	3.38	1.21787571			
5	−5.4	0.003736921	2.22	0.7975072			
6	0.8	0.003652301	1.56	0.44468582			
7	9.5	0.003539823	0.77	−0.26136476			
8	19.4	0.003419973	0.46	−0.77652879			
9	29.2	0.003309067	0.28	−1.27296568			
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							
28							
29							
30	R	1.98717					
31	Slope	10733.70	=5401.5*B30				
32	Intercept	4.3028E-09	=EXP(−19.264)				
33							



4. *Using the calculated coefficient of Arrhenius equation, we can write the equation for the viscosity value at 5°C as*

$$\mu = 4.3 \times 10^{-9}\, exp\left(\frac{5401}{5+273}\right)$$

$$= 1.178 \text{ Pa s}$$

5. *The viscosity at 5°C is 1.178 Pa s. This value is between the given values at temperatures 0.8° and 9.5°C.*

2.9 FLOW CHARACTERISTICS OF NON-NEWTONIAN FLUIDS

2.9.1 Properties of Non-Newtonian Fluids

It should be evident from the preceding discussion in this chapter that liquids offer interesting properties. They flow under gravity and do not retain their shape. They may exist as solids at one temperature, and liquid at another (e.g., ice cream and shortenings). Products such as applesauce, tomato purée, baby foods, soups, and salad dressings are

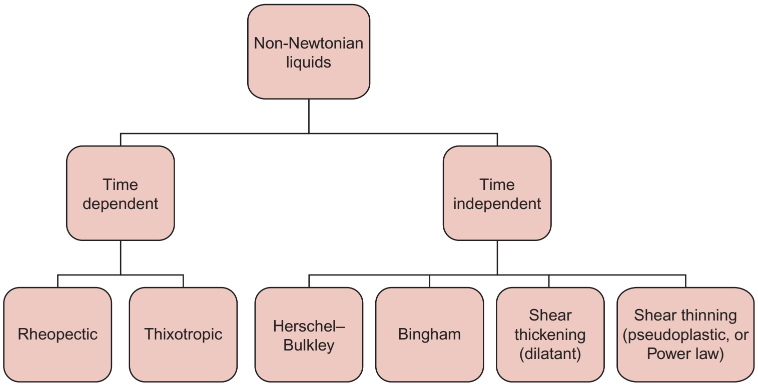


Figure 2.42 Classification of non-Newtonian liquids.

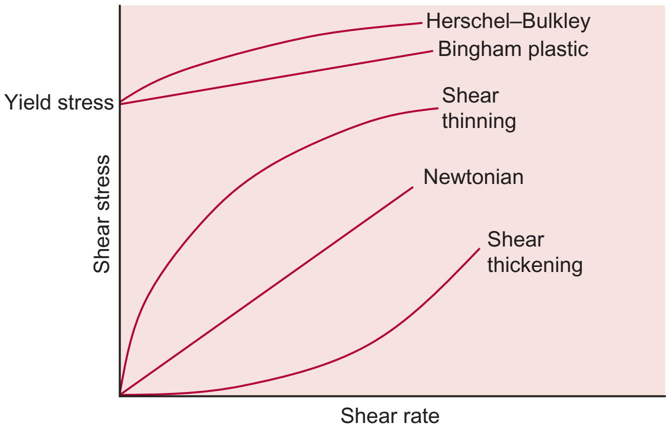
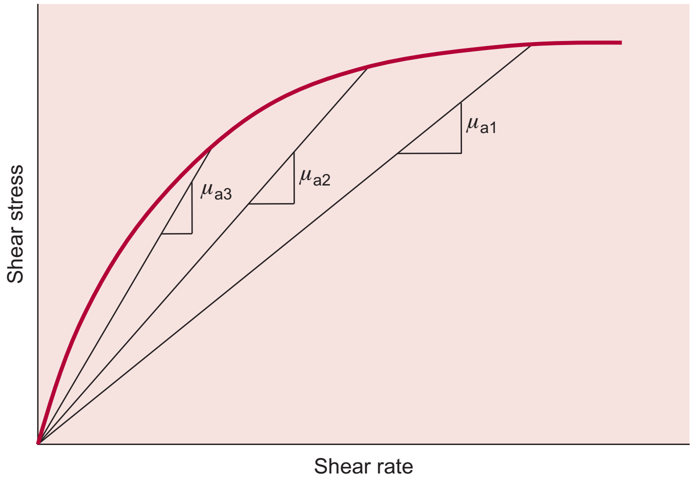


Figure 2.43 Relationship between shear stress and shear rate for Newtonian and non-Newtonian liquids.

suspensions of solid matter in liquid. When droplets of one liquid are submerged in another, we obtain emulsions—milk, for example.

The properties of non-Newtonian liquids can be classified as time-independent and time-dependent (Fig. 2.42). The **time-independent non-Newtonian liquids** respond immediately with a flow as soon as a small amount of shear stress is applied. Unlike Newtonian liquids, the relationship between shear stress and shear rate is nonlinear, as shown in Figure 2.43. There are two important types of time-independent non-Newtonian liquids, namely, shear-thinning liquids and shear-thickening liquids. The differences between these two types of liquids can be understood easily by considering another commonly used term, **apparent viscosity**.

An apparent viscosity is calculated by using a gross assumption that the non-Newtonian liquid is obeying Newton's law of viscosity



■ **Figure 2.44** Determination of apparent viscosity from plot of shear stress vs shear rate.

(Equation (2.10)). Thus, at any selected shear rate, a straight line is drawn from the selected point on the curve to the origin (Fig. 2.44). The slope of this straight line gives a value for the apparent viscosity. Using this method, it should be evident that the value obtained for apparent viscosity is dependent on the selected shear rate. Therefore the apparent viscosity must always be expressed along with the value of shear rate used to calculate it; otherwise, it is meaningless. For a **shear-thinning liquid**, as the shear rate increases, the apparent viscosity decreases; thus, the name shear thinning is used to describe the behavior of these liquids.

Shear-thinning liquids also are called **pseudoplastic** or **power law** liquids. Some common examples of shear-thinning liquids are condensed milk, fruit purées, mayonnaise, mustard, and vegetable soups. When shear-thinning products are shaken in a jar, they become more "fluid." Similarly, if these products are mixed at high intensity in a mixer, their viscosity decreases, which may aid in their mixing. There are several reasons to explain shear-thinning behavior. A liquid that appears homogenous to the naked eye may actually contain microscopic particulates submerged in it. When these liquids are subjected to a shear, the randomly distributed particles may orient themselves in the direction of flow; similarly, coiled particulates may deform and elongate in the direction of flow. Any agglomerated particles may break up into smaller particles. These types of modifications due to shearing action improve the flow of such fluids, and an increased "fluidity" is observed. They are also usually reversible. Thus, when the shearing

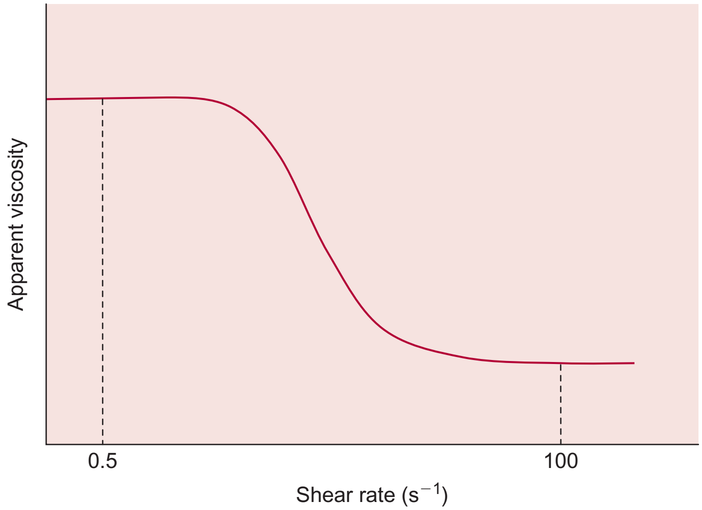


Figure 2.45 Apparent viscosity vs shear rate.

action is stopped, after a time lag, the particulates return to their original shape—the elongated particulates coil back, and separated particles may again agglomerate. Note that changes in viscosity at a very low shear rate ($\leq$0.5 s^{-1}) or a very high shear rate ($\geq$100 s^{-1}) are usually quite small, as seen in Figure 2.45. Therefore, in measuring rheological properties of power law fluids, a shear rate between 0.5 s^{-1} and 100 s^{-1} is used.

With some liquid foods, the processing steps may alter their flow properties. For example, raw egg at 21°C is a Newtonian fluid, but when frozen whole egg is thawed, its response changes to that of a shear-thinning liquid. Similarly, single-strength apple juice is a Newtonian liquid, but concentrated apple juice (depectinized and filtered) is a shear-thinning liquid.

If the increase in shear rate results in an increase in apparent viscosity, then the liquid is called a **shear-thickening liquid** (or sometimes referred to as **dilatant liquid**). Examples of shear-thickening liquids include 60% suspension of corn starch in water. With shear-thickening liquids, the apparent viscosity increases with increasing shear rate. These liquids become "stiffer" at higher shear rates. Mostly, these liquids are suspensions—solid particles in a liquid that acts as a plasticizer. At low shear rates, the liquid is sufficient to keep the solid particles well lubricated, and the suspension flows almost as a Newtonian liquid. But as shear rate increases, the solid particles begin to separate out, forming wedges while increasing the overall volume. Hence, they are called dilatant liquids. The liquid is then unable to act as a plasticizer. As a result, the overall suspension becomes more resistant to flow.

Another important class of non-Newtonian liquids requires the application of **yield stress** prior to any response. For example, certain types of tomato catsup will not flow until a certain yield stress is applied. For these types of liquids, a plot of shear stress against shear rate does not pass through the origin, as shown in Figure 2.43. After the application of yield stress, the response of these liquids can be similar to a Newtonian liquid; in that case, they are called **Bingham plastic**. On the other hand, if the response of a liquid, after the yield stress is applied, is similar to a shear-thinning flow, then these liquids are called **Herschel–Bulkley** fluids. These liquids that require a yield stress to flow may be viewed as having an interparticle or intermolecular network that resists low-level shear force when at rest. Below the yield stress, the material acts like a solid and does not flatten out on a horizontal surface due to force of gravity. It is only when the applied stress exceeds the forces holding the network together that the material begins to flow.

Time-dependent non-Newtonian liquids obtain a constant value of apparent viscosity only after a certain finite time has elapsed after the application of shear stress. These types of liquids are also called **thixotropic** materials; examples include certain types of starch pastes. For more discussion of these types of liquids, refer to Doublier and Lefebvre (1989).

A common mathematical model may be used to express the non-Newtonian characteristics. This model is referred to as the **Herschel–Bulkley model** (Herschel and Bulkley, 1926):

$$\sigma = K\left(\frac{du}{dy}\right)^n + \sigma_0 \tag{2.161}$$

where the values for the different coefficients are given in Table 2.4.

Another model, used in interpreting the flow data of chocolate, is the **Casson model** (Casson, 1959), given as

$$\sigma^{0.5} = \sigma_0^{0.5} + K(\dot{\gamma})^{0.5} \tag{2.162}$$

The square root of shear stress is plotted against the square root of shear rate to obtain a straight line. The slope of the line gives the consistency coefficient, and the square of the intercept gives the yield stress.

Table 2.4 Values of Coefficients in Herschel—Bulkley Fluid Model

Fluid	K	N	σ_0	Typical examples
Herschel—Bulkley	>0	$0 < n < \infty$	>0	minced fish paste, raisin paste
Newtonian	>0	1	0	water, fruit juice, honey, milk, vegetable oil
Shear-thinning (pseudoplastic)	>0	$0 < n < 1$	0	applesauce, banana purée, orange juice concentrate
Shear-thickening	>0	$1 < n < \infty$	0	some types of honey, 40% raw corn starch solution
Bingham plastic	>0	1	>0	toothpaste, tomato paste

Source: Steffe (1996)

Example 2.22

Rheological data for Swedish commercial milk chocolate at 40°C is shown in the following. Using the Casson model, determine the consistency coefficient and the yield stress.

Shear rate	Shear stress
0.099	28.6
0.14	35.7
0.199	42.8
0.39	52.4
0.79	61.9
1.6	71.4
2.4	80.9
3.9	100
6.4	123.8
7.9	133.3
11.5	164.2
13.1	178.5
15.9	201.1
17.9	221.3
19.9	235.6

Given

Data on shear rate and shear stress given in the preceding table.

Approach

We will use a spreadsheet for this example.

Solution

1. *We will use Equation (2.162). Using a spreadsheet, first enter the shear stress and shear rate data into columns A and B. Then develop*

Figure E2.8 Spreadsheet solution for Example 2.22.

	A	B	C	D	E
1	Shear rate	Shear stress	Shear rate^0.5	Shear stress^0.5	
2	0.099	28.6	0.315	5.348	
3	0.14	35.7	0.374	5.975	
4	0.199	42.8	0.446	6.542	
5	0.39	52.4	0.624	7.239	
6	0.79	61.9	0.889	7.868	
7	1.6	71.4	1.265	8.450	
8	2.4	80.9	1.549	8.994	
9	3.9	100	1.975	10.000	
10	6.4	123.8	2.530	11.127	
11	7.9	133.3	2.811	11.546	
12	11.5	164.2	3.391	12.814	
13	13.1	178.5	3.619	13.360	
14	15.9	201.1	3.987	14.181	
15	17.9	221.3	4.231	14.876	
16	19.9	235.6	4.461	15.349	
17					
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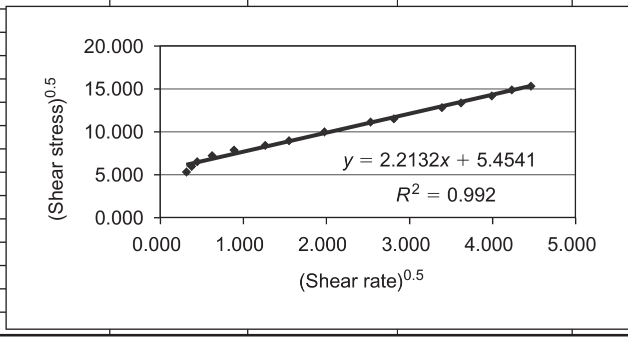
30					
31					



new columns, C and D, with square root of shear stress and shear rate. Then plot the data from columns C and D as a scatter plot, as shown in Figure E2.8. If using Excel, then use "trend line" to determine the slope, intercept, and data fit.

2. *The slope is 2.213, and intercept is 5.4541.*
3. *The consistency coefficient is*

$$K = 2.213\ Pa^{0.5}\ s^{0.5}$$

and

$$\text{Yield stress } \sigma_0 = 5.4541^2 = 29.75\ Pa$$

4. *These coefficients were obtained when the entire range of shear rate $0-20\ s^{-1}$ was used. For further analysis of these data see Steffe (1996).*

2.9.2 Velocity Profile of a Power Law Fluid

We recall from Equation (2.161) that the shear stress and shear rate are related by a power law equation, as follows:

$$\sigma = K\left(-\frac{du}{dr}\right)^n \tag{2.163}$$

Note that du/dr is negative in a pipe flow where the velocity decreases from the center to the surface. Therefore, a negative sign is used to make the shear stress positive.

From Equations (2.27) and (2.163)

$$K\left(-\frac{du}{dr}\right)^n = \frac{\Delta P r}{2\,L} \tag{2.164}$$

Rearranging and setting up integrals from the central axis to the wall of the pipe,

$$-\int_u^0 du = \left(\frac{\Delta P}{2LK}\right)^{1/n} \int_r^R r^{1/n}\, dr \tag{2.165}$$

Integrating between the limits,

$$u(r) = \left(\frac{\Delta P}{2LK}\right)^{1/n} \left(\frac{n}{n+1}\right) \left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \tag{2.166}$$

Equation (2.166) gives the velocity profile for a power law fluid.

2.9.3 Volumetric Flow Rate of a Power Law Fluid

The volumetric flow rate is obtained by integrating the velocity profile for a ring element as previously discussed for Newtonian fluid in Section 2.3.4:

$$\dot{V} = \int_{r=0}^{r=R} u(r) 2\pi r dr \tag{2.167}$$

Substituting Equation (2.166),

$$\dot{V} = \left(\frac{\Delta P}{2LK}\right)^{1/n} \left(\frac{n}{n+1}\right) 2\pi \int_{r=0}^{r=R} \left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) r dr \tag{2.168}$$

Integrating between limits,

$$\dot{V} = \left(\frac{\Delta P}{2LK}\right)^{1/n} \left(\frac{n}{n+1}\right) 2\pi \left| \frac{r^2}{2} R^{\frac{n+1}{n}} - \frac{r^{\frac{2n+1}{n}+1}}{\frac{2n+1}{n}+1} \right|_0^R \tag{2.169}$$

Evaluating the limits,

$$\dot{V} = \left(\frac{\Delta P}{2LK}\right)^{1/n} 2\pi\left(\frac{n}{n+1}\right)\left[\frac{R^{\frac{3n+1}{n}}}{2} - \frac{R^{\frac{3n+1}{n}}}{\frac{3n+1}{n}}\right] \tag{2.170}$$

and simplifying,

$$\dot{V} = \pi\left(\frac{n}{3n+1}\right)\left(\frac{\Delta P}{2LK}\right)^{1/n} R^{\frac{3n+1}{n}} \tag{2.171}$$

Equation (2.171) gives the volumetric flow rate for a power law fluid.

2.9.4 Average Velocity in a Power Law Fluid

Recall from Equation (2.17) that

$$\bar{u} = \frac{\dot{V}}{\pi R^2} \tag{2.172}$$

Substituting Equation (2.171) in (2.172) we obtain the average velocity:

$$\bar{u} = \left(\frac{n}{3n+1}\right)\left(\frac{\Delta P}{2LK}\right)^{1/n} R^{\frac{n+1}{n}} \tag{2.173}$$

For Newtonian fluids the velocity ratio, $u/u_{\max}$, was given by Equation (2.42) (for laminar flow), or Equation (2.48) (for turbulent flow). For non-Newtonian fluids, the velocity ratio can be obtained from Figure 2.46 based on the work of Palmer and Jones (1976).

2.9.5 Friction Factor and Generalized Reynolds Number for Power Law Fluids

In Section 2.3.5, we defined Fanning friction factor as

$$f = \frac{\Delta P D}{2\rho L\bar{u}^2} \tag{2.174}$$

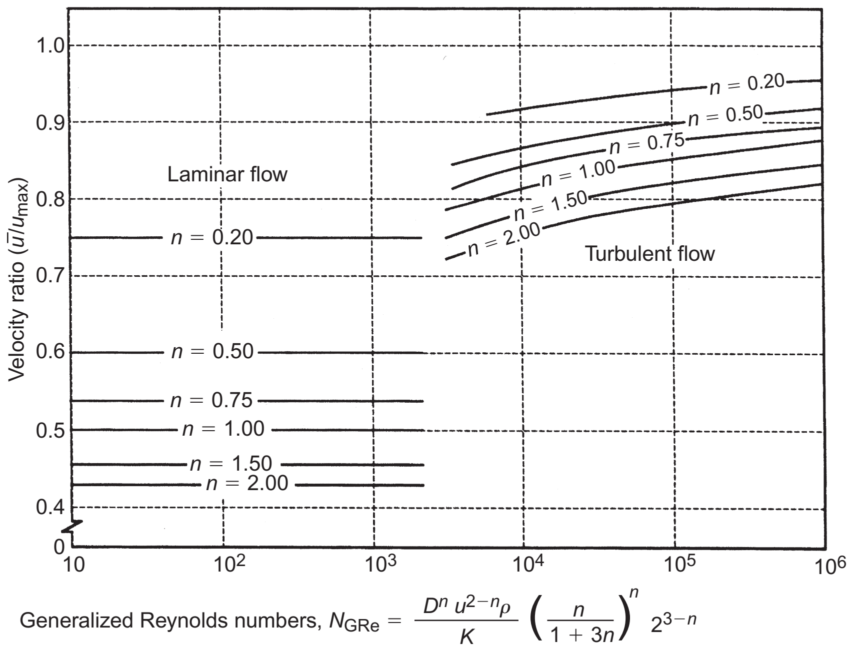


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Figure 2.46 Plot of velocity ratio vs generalized Reynolds numbers. (From Palmer and Jones, 1976)

We can rearrange the terms in Equation (2.173) as

$$\frac{\Delta P}{L} = \frac{4K\bar{u}^n}{D^{n+1}} \left(\frac{6n+2}{n}\right)^n \tag{2.175}$$

Substituting in Equation (2.174),

$$f = \frac{2K}{\rho \bar{u}^{2-n} D^n} \left(\frac{6n+2}{n}\right)^n \tag{2.176}$$

Similar to Equation (2.53), the Fanning friction factor for laminar flow for a power law fluid is

$$f = \frac{16}{N_{GRe}} \tag{2.177}$$

where by comparison of Equation (2.176) and Equation (2.177), we obtain the generalized Reynolds number as

$$N_{\mathrm{GRe}} = \frac{8D^n \bar{u}^{2-n} \rho}{K} \left(\frac{n}{6n+2} \right)^n \tag{2.178}$$

or, rearranging terms,

$$N_{\mathrm{GRe}} = \frac{D^n \bar{u}^{2-n} \rho}{K 8^{n-1}} \left(\frac{4n}{3n+1} \right)^n \tag{2.179}$$

Note that if we substitute the values $n = 1$ and $K = \mu$, Equation (2.179) reduces to the Reynolds number for Newtonian fluids.

Example 2.23 illustrates the use of various expressions obtained in this section for the power law fluids.

Example 2.23

A non-Newtonian fluid is flowing in a 10-m-long pipe. The inside diameter of the pipe is 3.5 cm. The pressure drop is measured at 100 kPa. The consistency coefficient is 5.2 and flow behavior index is 0.45. Calculate and plot the velocity profile, volumetric flow rate, average velocity, generalized Reynolds number, and friction factor.

Given

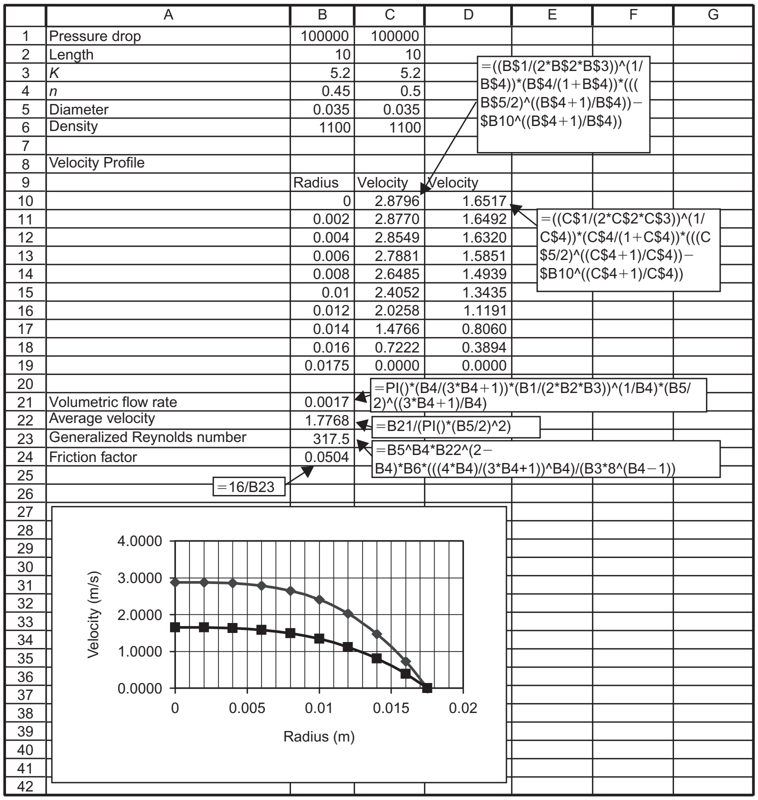
Pressure drop = 100 kPa
Inside diameter = 3.5 cm = 0.035 m
Consistency coefficient = 5.2
Flow behavior index = 0.45
Length = 10 m

Approach

We will program a spreadsheet with the given data and equations from Section 2.9 to obtain the plot and the results.

Solution

1. *The solution is shown in the spreadsheet. Note that as the flow behavior index increases, the velocity at the center decreases. Once a spreadsheet is programmed as shown in Figure E2.9, other values of pressure drop, consistency coefficient, and flow behavior index may be substituted in the appropriate cells to observe their influence on the calculated results. For example an increase in flow behavior index from 0.45 to 0.5 results in a significant change in the velocity profile, as shown in Figure E2.9.*



	A	B	C	D	E	F	G
1	Pressure drop	100000	100000				
2	Length	10	10				
3	K	5.2	5.2				
4	n	0.45	0.5				
5	Diameter	0.035	0.035				
6	Density	1100	1100				
7							
8	Velocity Profile						
9		Radius	Velocity	Velocity			
10		0	2.8796	1.6517			
11		0.002	2.8770	1.6492			
12		0.004	2.8549	1.6320			
13		0.006	2.7881	1.5851			
14		0.008	2.6485	1.4939			
15		0.01	2.4052	1.3435			
16		0.012	2.0258	1.1191			
17		0.014	1.4766	0.8060			
18		0.016	0.7222	0.3894			
19		0.0175	0.0000	0.0000			
20							
21	Volumetric flow rate	0.0017					
22	Average velocity	1.7768					
23	Generalized Reynolds number	317.5					
24	Friction factor	0.0504					

Figure E2.9 Spreadsheet solution for Example 2.23.

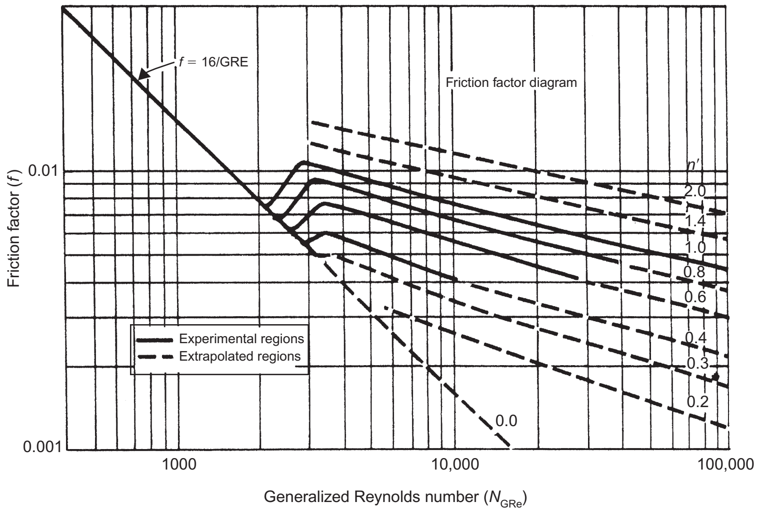
2.9.6 Computation of Pumping Requirement of Non-Newtonian Liquids

The computation of pumping requirements of non-Newtonian liquids is similar to that shown earlier for Newtonian liquids in Section 2.5.5. Equation (2.95) is modified to incorporate the non-Newtonian properties as follows:

$$E_P = \frac{P_2 - P_1}{\rho} + \frac{1}{2\alpha'}(u_2^2 - u_1^2) + g(z_2 - z_1) + \frac{2f\bar{u}^2 L}{D} + C'_{fe}\frac{\bar{u}^2}{2} + C'_{fc}\frac{\bar{u}^2}{2} + C'_{ff}\frac{\bar{u}^2}{2} \tag{2.180}$$

where, for laminar flow and shear-thinning liquids,

$$\alpha' = \frac{(2n+1)(5n+3)}{3(3n+1)^2} \tag{2.181}$$



■ **Figure 2.47** Friction factor-Generalized Reynolds number chart for non-Newtonian flow in cylindrical tubes. (From Dodge and Metzner, 1959)

and for turbulent flow

$$\alpha' = 1 \tag{2.182}$$

The friction factor, f, for laminar flow is given by Equation (2.177) and for turbulent flow, f is obtained from Figure 2.47.

For coefficients C'_{fe}, C'_{fc}, and C'_{ff}, experimental data for non-Newtonian fluids are limited. The following rule-of-thumb guidelines are proposed by Steffe (1996) to determine the three coefficients for non-Newtonian fluids.

a. For non-Newtonian fluids above a generalized Reynolds number of 500, use data for Newtonian fluids in turbulent flow (see Table 2.2 and Equations (2.90) and (2.92)).

b. For non-Newtonian fluids, for the range $20 < N_{GRe} < 500$, first determine coefficients C_{fe}, C_{fc}, C_{ff} as in step (a), then use the following expressions:

$$C'_{fe} = \frac{500 \times C_{fe}}{N_{GRe}}; \quad C'_{fc} = \frac{500 \times C_{fc}}{N_{GRe}}; \quad C'_{ff} = \frac{500 \times C_{ff}}{N_{GRe}} \tag{2.183}$$

The resulting values of the coefficients are then substituted in Equation (2.180).

Example 2.24 illustrates the determination of pumping requirements of a non-Newtonian liquid.

Example 2.24

A non-Newtonian fluid is being pumped from one tank to another in a 0.0348-m-diameter pipe with a mass flow rate of 1.97 kg/s. The properties of the fluid are as follows: density 1250 kg/m^3, consistency coefficient 5.2 $P_a s^n$, flow behavior index 0.45. The total length of pipe between the tanks is 10 m. The difference of elevation from inlet to exit is 3 m. The fittings include three long-radius 90° flanged elbows, and one fully open angle valve. Furthermore, a filter present in the pipeline causes a 100-kPa pressure drop. Set up a spreadsheet to calculate the pumping requirements.

Given

Pipe diameter = 0.0348 m
Mass flow rate = 1.97 kg/s
Density = 1250 kg/m^3
Consistency coefficient = 5.2 Pa s^n
Flow behavior index = 0.45
Length of pipe = 10 m
Difference in elevation = 3 m
Pressure drop in filter = 100 kPa
C_{ff} of 90° flanged long radius elbows from Table 2.3 = 0.2
C_{ff} of fully open angle valve from Table 2.3 = 2

Approach

We will develop a spreadsheet using the given data and appropriate equations to calculate velocity from the mass flow rate, generalized Reynolds number, correction factor, α, coefficients C_{fe} and C_{ff} for various fittings, and friction factor. Finally we will substitute the calculated values in Equation (2.180) to determine the pumping requirements.

Solution

1. *Enter given data as shown in A1:B12 in Figure E2.10.*
2. *Enter the following equations to calculate required values.*

Equation number	To calculate	Cell
(2.15)	velocity	B15
(2.179)	generalized Reynolds number	B16
(2.181)	correction factor	B17
(2.177)	friction factor	B21
(2.183)	coefficients for fittings, contraction	B24
(2.180)	energy for pumping	B25

3. *The spreadsheet may be used to calculate pumping requirements for different input values.*

■ **Figure E2.10** Spreadsheet solution for Example 2.24.

	A	B	C	D	E	F	G
1	Given						
2	Pipe diameter (m)	0.03					
3	Mass flow rate (kg/s)	2					
4	Density (kg/m3)	1250					
5	Pressure drop (kPa)	100					
6	Angle valves	1					
7	Long radius elbows	3					
8	Length (m)	10					
9	Change in elevation (m)	3					
10	*K* (Pa s^n)	5.2					
11	*n*	0.45					
12	Pump efficiency	0.85					
13							
14							
15	Velocity	2.26	← =B3/(B4*(PI()*B2^2/4))				
16	Generalized Reynolds number	489.92	← =(B2^B11*B15^(2-B11)*B4/(B10*8^(B11−1)))*(4*B11/(3*B11=1))^B11				
17	Correction factor	1.20	← =2*(2*B11+1)*(5*B11+3)/(3*(3*B11+1)^2)				
18	C_{fe} entrance	0.5					
19	C_{ff} long radius elbow	0.2					
20	C_{ff} angle valve	2					
21	Friction factor	0.0327	← =16/B16				
22	Friction Loss (J/kg)	8.11	← =(B18*500/B16+3*B19*500/B16+B20*500/B16)*B15^2/2				
23	Friction loss due to filter	80.00	← =B5*1000/B4				
24	Total friction loss	88.11	← =B22+B23				
25	Energy for pump (J/kg)	233.34	← =9.81*B9+B15^2/B17+2*B21*B15^2*B8/B2+B24				
26	Power requirement (W)	549.04	← =B26*B3/B12				
27							
28							

2.10 TRANSPORT OF SOLID FOODS

Many food products and ingredients are transported throughout a processing plant in the form of a solid, as opposed to liquids. The handling, storage, and movement of these materials require different types of equipment and process design. Many of the considerations are similar to liquids, in terms of requirements for sanitary design of product-contact surfaces within equipment and the attention to ensuring that designs reduce impact on product quality attributes. Ultimately, the transport of solid pieces, granular products, and/or food powders requires power inputs to an operation designed specifically for the product being handled. In many situations, gravity becomes a contributor to the transport, and introduces unique factors into the transport challenges.

The computation of material transport requirements depend directly on properties of the granular solid or powder. These properties include consideration for friction at the interface between the solid material and the surface of the transport conduit. After establishment of appropriate property magnitudes, the power requirements or similar process design configuration can be estimated.

2.10.1 Properties of Granular Materials and Powders

The physical properties of granular materials and powders have direct influence on the transport of these types of food within a food processing operation. In the following sections, the basic relationships used to predict or measure the properties of these types of food product will be presented and described.

2.10.1.1 *Bulk Density*

For granular materials or powders, there are different types of density to be described. As indicated in Section 1.5, bulk density can be defined as follows:

$$\rho_B = \frac{m}{V} \tag{2.184}$$

or the overall mass of the material divided by the volume occupied by the material. Depending on other characteristics of the food material, this property may require more specific description. For example, food powders will occupy variable volume depending on the size of individual particles and the volume of the space between the particles. Given these observations, the magnitude of the bulk density will vary with the extent of packing within the food particle structure. One approach to measurement includes *Loose Bulk Density*, obtained by careful placement of the granular material in a defined volumetric space without vibration, followed by measurement of the mass. *Packed Bulk Density* would be measured by vibration of a defined mass until the volume is constant, then compute bulk density according to Equation (2.184). In practice, bulk density will vary with the conditions within the operation, but the magnitude should fall between the extremes indicated by the measurement approach.

The bulk density can be predicted by the following relationship:

$$\rho_B = \in_p \rho_p + \in_a \rho_a \tag{2.185}$$

where $\in_p$ is the volume fraction occupied by particles and $\in_a$ is the volume fraction occupied by air. The parameter referred to as void (υ) or Interparticle Porosity (see Eq. 1.2) is defined as the ratio of volume of space not occupied by particles ($\in_a$) to the total volume. The following expression would apply:

$$\upsilon = 1 - \left(\frac{\rho_b}{\rho_p}\right) \tag{2.186}$$

and would reflect the variability in bulk density discussed in the previous paragraph.

2.10.1.2 *Particle Density*

As indicated in Section 1.5, the density of individual food particles is referred to as particle density, and is a function of gas phase (air) volume trapped within the particle structure. This property is measured by a picnometer; using a known-density solvent to replace the gas within the particle structure.

The particle density is predicted from the following relationship:

$$\rho_p = \rho_s e_{so} + \rho_a e_a \tag{2.187}$$

where the density of the gas phase or air (ρ_α) is obtained from standard tables (Table A.4), and the density of particle solids is based on product composition and the coefficients in Table A.2.9. The volume fractions of solids (e_{so}) and gas phase or air (e_a) would be evident in magnitude of the particle density.

Example 2.25

Estimate the particle density of a nonfat dry milk particle at 20°C, when 10% of the particle volume is air space and the moisture content is 3.5% (wet basis).

Solution

1. *Based on information presented in Table A.2, the composition of nonfat dry milk is 35.6% protein, 52% carbohydrate, 1% fat, 7.9% ash, and 3.5% water.*
2. *Using the relationships in Table A.2.9, the density of product components at 20°C are:*

   Protein = 1319.5 kg/m^3
   Carbohydrate = 1592.9 kg/m^3
   Fat = 917.2 kg/m^3
   Ash = 2418.2 kg/m^3
   Water = 995.7 kg/m^3
   Air = 1.164 kg/m^3 (from Table A.4.4)

3. *The density of the particle solids is a function of the mass of each component:*

$$\begin{aligned}\rho_{SO} &= (0.356)(1319.5) + (0.52)(1592.9) + (0.01)(917.2) \\ &\quad + (0.079)(2418.2) + (0.035)(995.7) \\ &= 1533.1 \text{ kg/m}^3\end{aligned}$$

4. *The particle density is estimated from Equation (2.187):*

$$\rho_p = (1533.1)(0.9) + (1.164)(0.1) = 1379.9 \text{ kg/m}^3$$

As indicated by the solution, the particle density is influenced primarily by the particle solids.

Porosity (Ψ) is the ratio of the air volume to total volume occupied by the product powder. As introduced in Section 1.5, the relationship of porosity to bulk density becomes:

$$\Psi = 1 - \left(\frac{\rho_b}{\rho_{so}}\right) \tag{2.188}$$

and the relationship to particle density is given by Equation (2.187).

All the properties of granular materials and powders are functions of moisture content and temperature. These relationships are accounted for in the dependence of individual components on moisture content and temperature, but other changes with the particle structure will occur. Details of these changes are discussed by Heldman (2001).

2.10.1.3 *Particle Size and Size Distribution*

An important property of a granular food or powder is particle size. This property has direct impact on the magnitude of the bulk density, as well as the porosity. Particle sizes are measured by using several different techniques, including a sieve, a microscope, or a light-scattering instrument (Coulter Counter). The results of such measurements reveal that a range of particle sizes exists within the typical food powder or granular material. These observations emphasize the importance of using at least two parameters to describe the particle size properties; a mean diameter and a standard deviation.

Mugele and Evans (1951) suggested a systematic approach to description of particle sizes for granular materials. The approach suggests the following model:

$$d_{qp}^{q-p} = \frac{\sum (d^q N)}{\sum (d^p N)} \tag{2.189}$$

where d is the particle diameter, N is the number of particles with a given diameter, and q and p are parameters defined in Table 2.5. For example, the arithmetic or linear mean diameter would become:

$$d_L = \frac{\sum dN}{N} \tag{2.190}$$

Table 2.5 Notations of Mean Particle Size for Use in Equation (2.189)

Symbol	Name of Mean Diameter	*P*	*q*	Order
x_L	Linear (arithmetic)	0	1	1
x_s	Surface	0	2	2
x_v	Volume	0	3	3
x_m	Mass	0	3	3
x_{sd}	Surface-diameter	1	2	3
x_{vd}	Volume-diameter	1	3	4
x_{vs}	Volume-surface	2	3	5
x_{ms}	Mass-surface	3	4	7

Source: *Mugele and Evans (1951)*

where the value is the straight forward mean of particle diameters within the distribution. Another commonly used expression is referred to as the Sauter mean diameter:

$$d_{vs} = \frac{\sum d^3 N}{\sum d^2 N} \tag{2.191}$$

and expresses the mean diameter as a function of the ratio of particle volume to particle surface area.

Most often, the distribution of particle sizes in a granular food or powder is described by log-normal density function. The parameters include the arithmetic log-geometric mean:

$$\ln d_g = \frac{\sum (N \ln d)}{N} \tag{2.192}$$

and the geometric standard deviation, as follows:

$$\ln s_{dg} = \left\{ \frac{\sum [N(\ln d - \ln d_g)^2]}{N} \right\}^{1/2} \tag{2.193}$$

These parameters are estimated from measurement of particle size, and the number of particles in sizes ranges over the range of sizes in the granular food or powder.

Example 2.26

The particle size distribution of a sample dry coffee product has been measured, and the results are as follows:

Portion (%)	Particle size (micron)
2	40
8	30
50	20
40	15
10	10

Estimate the mean particle size based on the ratio of volume to surface area.

Solution

1. *Given Equation (2.191):*

$$d_{vs} = \frac{(40)^3(2) + (30)^3(8) + (20)^3(50) + (15)^3(40) + (10)^3(10)}{(40)^2(2) + (30)^2(8) + (20)^2(50) + (15)^2(40) + (10)^2(10)}$$

2. *The volume/surface area or Sauter mean particle size diameter is:*

$$d_{vs} = 22 \text{ micron}$$

2.10.1.4 *Particle Flow*

The movement or flow of a granular food or powder is influenced by several different properties of the powder, including several of the particle properties previously described. A property of the granular food that is directly related to flow is *angle of repose*. This is a relatively simple property to measure by allowing the powder to flow from a container over a horizontal surface. The height (H) of the mound of powder and the circumference of the mound (S) are measured, and the angle of repose is computed using the following equation:

$$\tan\beta = \frac{2\pi H}{S} \tag{2.194}$$

The magnitude of this property will vary with density, particle size, moisture content, and particle size distribution of the granular powder.

A more fundamental property of powder flow is the angle of internal friction. This property is defined by the following expression:

$$\tan\varphi = \frac{\sigma}{\sigma_n} \tag{2.195}$$

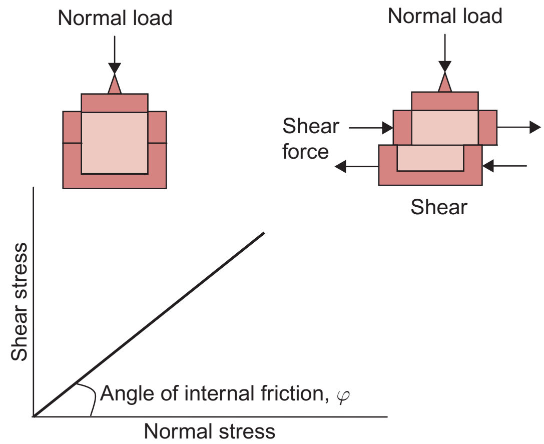


Figure 2.48 The relationship of shear stress to normal stress for a noncohesive powder.

indicating that the angle of internal friction (φ) is proportional to a ratio of the shear stress (σ) to the normal stress (σ_n). The magnitude of this property is determined by measurement of the forces required to move one layer of powder over another layer at various magnitudes of normal stress, as indicated in Figure 2.48.

A fundamental approach to measurement of granular material flow properties is based on a method proposed by Jenike (1970). Measurements of shear stress and normal stress are obtained using a specific shear cell, but are used to generate parameters, such as the unconfined yield stress (f_c) and the major unconsolidated stress (σ_1). These two parameters are then used to define the flow function (f'_c), as follows:

$$f'_c = \frac{\sigma_1}{f_c} \tag{2.196}$$

This property (f'_c) characterizes the flowability of powders, and with specific applications in the design of bins and hoppers for granular foods. Details on measurement of the flow function (f'_c) are provided by Rao (2006). The relationships of the magnitude of the flow function (f'_c) to flowability of powders are presented in Table 2.6.

2.10.2 Flow of Granular Foods

A significant application of food powder flow properties is the description of product flow from a storage vessel or container. The properties of the granular material have direct influence on the design of the vessel, and specifically the configuration of the vessel at the

Table 2.6 Flowability of Powders According to Jenike's Flow Function

Magnitude of flow function f'_c	Flowability of powders	Type of powder
$f'_c < 2$	Very cohesive, nonflowing	Cohesive powders
$2 < f'_c < 4$	Cohesive	
$4 < f'_c < 10$	Easy flowing	Noncohesive powders
$10 < f'_c$	Free flowing	

exit opening. The mass flow rate of a granular food from a storage vessel can be estimated from the following expression:

$$m_g = \frac{C_g \pi \rho_b \left(\frac{D^5 g \tan \beta}{2}\right)^{0.5}}{4} \tag{2.197}$$

where the mass flow rate (m_g) is a function of bulk density of the powder (ρ_b), diameter of the opening at the exit from the vessel (D), the angle of repose for the powder (β), and a discharge coefficient (C_g). The discharge coefficients will vary in magnitude with container configuration; usually between 0.5 and 0.7. Equation (2.197) is limited to situations where the ratio of particle diameter to opening diameter is less than 0.1.

A critical problem in the use of gravity to induce granular flow from a storage vessel is the tendency of arching or bridging to occur. This blockage of flow from the container is directly related to the powder properties, specifically the density and angle of internal friction. A closely related concern during gravity flow from a storage container is the occurrence of plug flow as opposed to mass flow, as illustrated in Figure 2.49. As illustrated, plug flow results in the collection of powder in regions of the container near the outlet, and the material is not removed without assistance. The vessel design feature is the cone angle (θ_c), or the angle between the vessel surface leading to the exit and the vertical sides of the container. This angle is dependent on the angle of internal friction for the powder.

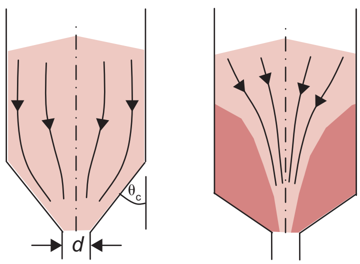


Figure 2.49 An illustration of food powder flow from a storage vessel.

An expression proposed to assist in the design of the outlet from a storage vessel for a granular food is as follows:

$$D_b = \left(\frac{C_b}{\rho_b}\right)[1 + \sin \varphi] \tag{2.198}$$

where D_b is the minimum diameter of opening needed to prevent bridging of the powder above the opening, and is dependent on the natural

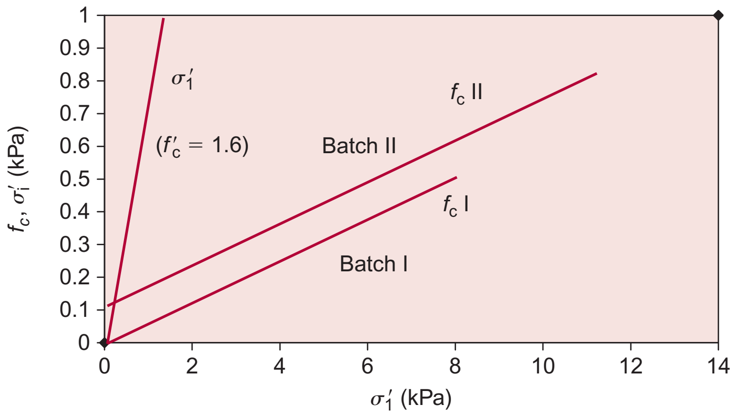


Figure 2.50 The relationship of shear stress to normal stress for instant cocoa powder. (From Schubert, H. (1987a). *J. Food Eng.* 6: 1—32; Schubert, H. (1987b). *J. Food Eng.* 6: 83—102. With permission.)

cohesiveness parameter (C_b) as determined during measurement of the angle of internal friction. More specifically, the natural cohesiveness parameter is the intercept on the shear stress axis of the shear stress versus normal stress plot, as illustrated in Figure 2.50. The angle of internal friction (φ) is the angle of the shear stress (τ) to the normal stress (σ).

Example 2.27

Estimate the minimum diameter of an opening from a storage vessel for an instant cocoa powder. The bulk density of the food powder is 450 kg/m^3.

Solution

1. *Based on the measurements of shear stress versus normal stress presented in Figure 2.50, the angle of internal friction (for Batch II) is determined by using Equation (2.195) to obtain:*

$$\tan\varphi = \frac{(1-0.11)}{14} = 0.064$$
$$\varphi = 3.66^\circ$$

2. *The same measurements indicate that the natural cohesiveness parameter (C_b) for the same batch (based on the intercept on the shear stress axis) is:*

$$C_b = 0.11 \text{ kPa} = 110 \text{ Pa}$$

3. *Using Equation (2.198):*

$$D_b = \left(\frac{110}{450}\right)[1+\sin(3.66)]$$
$$D_b = 0.26 \text{ m} = 26 \text{ cm}$$

4. *The result indicates that the diameter of the outlet for the storage vessel would need to exceed 26 cm to avoid bridging for this product.*

A more thorough analysis of the granular flow from a storage container involves use of the flow function (f'_c) in Equation (2.196). Based on this approach, the minimum diameter to avoid arching (D_b) is estimated from:

$$D_b = \frac{\sigma' H(\theta)}{g\rho_b} \tag{2.199}$$

where $H(\theta)$ is a function of the geometry of the exit region from the storage vessel (see Jenike, 1970). The major principle stress (σ') acting in the arch or bridge can be estimated from:

$$\sigma' = \frac{\sigma_1}{f'_c} \tag{2.200}$$

where σ_1 is the unconsolidated stress involved in measurement of the flow function (f'_c). Once flowability of the powder has been established, Table 2.6 can be used to estimate the flow function (f'_c) and the minimum diameter of the opening from the granular food storage vessel from Equation (2.199).

2.11 PROCESS CONTROLS IN FOOD PROCESSING

A typical food processing factory involves a number of unit operations that are carried out with different processing equipment: pumping, mixing, heating, cooling, freezing, drying, and packaging. Often the processing equipment operates in a continuous mode, which results in higher processing efficiencies than the batch mode. In designing a food processing plant, the processing equipment is arranged in a logical manner so that as raw food enters the plant, it is conveyed from one piece of equipment to the next while undergoing the desired processing.

Figure 2.51 is a flow diagram of typical operations used in the manufacture of canned tomatoes. The operations include unloading tomatoes from the trucks, washing, cleaning, grading, peeling, filling into cans, and retorting. Most of the equipment used to carry out these processing operations is linked with different types of conveyors that allow the entire process to proceed in a continuous system. Some of the operations require human intervention, namely, inspecting incoming tomatoes to ensure that any undesirable foreign objects such as dirt clods and severely damaged fruit are removed. However, most of the processing equipment is operated without much human intervention with the use of automatic controllers and sensors.

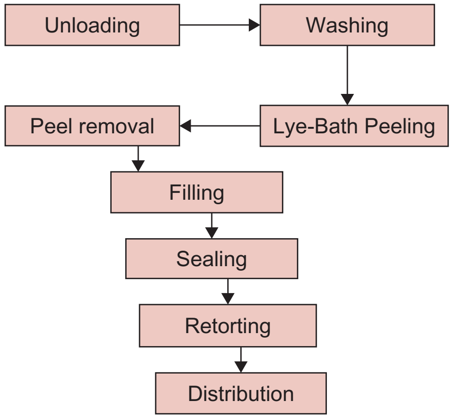


Figure 2.51 Manufacturing steps in canning tomatoes.

The following criteria must be addressed in food manufacturing:

- Required production capacity
- Quality and hygiene of the end product
- Flexible manufacturing
- Optimum use of labor
- Economical operation that maximizes profit
- Compliance with the environmental regulations imposed by the local, national, or international laws
- Providing a safe working environment
- Meeting any special constraints imposed by the processing equipment.

These criteria require that food processing operations should be monitored continually and any deviations are promptly addressed.

In this section, we will examine some of the underlying principles of automatic control of processing equipment. The design and implementation of process controls requires advanced mathematics, which is beyond the scope of this book. However, you will be introduced to some of the common terminology and approaches used in designing these controls.

Let us consider a simple case where the task assigned to an operator is to pump wine into different tanks (Fig. 2.52). For this task, the operator uses certain logic: choosing a precleaned empty tank, opening a valve to direct wine to that tank, knowing when it is full, and then directing wine to the next available tank, and so on. The operator keeps a check on the level of wine in the tank being filled to ensure

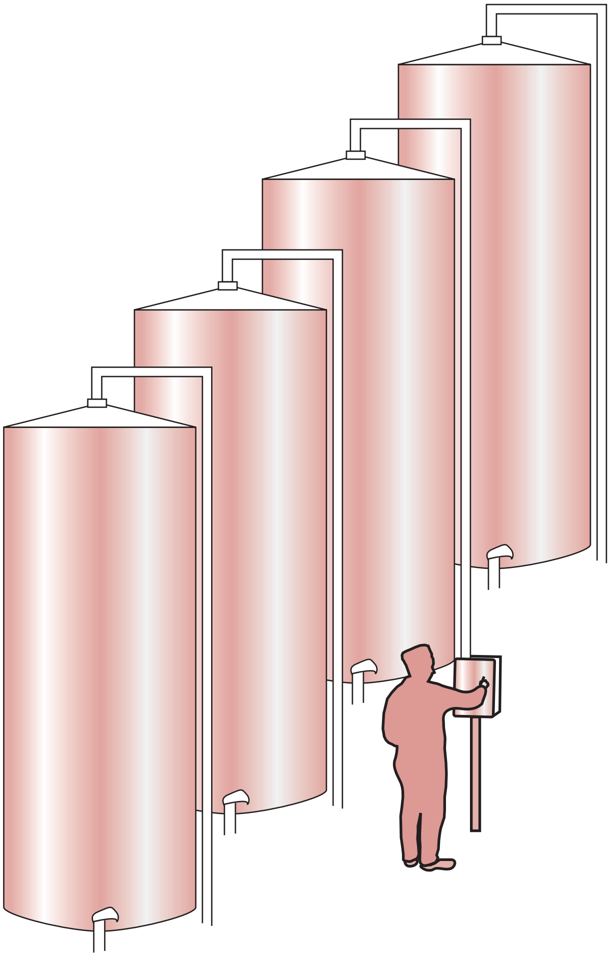


Figure 2.52 Manual control of tank filling operation in a winery.

that it does not overfill and cause loss of product. A similar logic may be programmed into an automatic control system to accomplish the desired task without significant human intervention.

2.11.1 Processing Variables and Performance Indicators

In operating food processing equipment, an operator is often concerned with a variety of different process variables. For example, in a heating system, the temperature of a product may require careful monitoring. When milk is pasteurized using a heat exchanger, the temperature of the milk must reach 71°C and be held there for

16 seconds to destroy harmful pathogens. Therefore, the operator must ensure that the temperature reaches the desired value for the specified time or the milk will be either underprocessed (resulting in an unsafe product) or overprocessed with impairment to quality. Similarly, in operating different processing equipment, the flow rate, level, pressure, or weight may be an important variable that requires careful monitoring and control.

Controlled variables are simply those variables that can be controlled in a system. For example, the steam composition, steam flow rate, temperature of a water stream, and level of water in a tank are all variables that can be controlled. When heating milk, temperature is a controlled variable. Other examples of a controlled variable include pressure, density, moisture content, and measurable quality attributes such as color.

Uncontrolled variables are those variables that cannot be controlled when a process is carried out. For example, during operation of an extruder, the impact of operation on the extruder screw surface is an uncontrolled variable.

Manipulated variables are dependent variables that can be changed to bring about a desired outcome. For example, by changing the flow rate of steam to a tank of water, the water temperature will change. This variable may be manipulated by either a human operator or a control mechanism. When heating water in a chamber, the feed rate of water is a manipulated variable. A measured variable is used to alter the manipulated variable. Examples of measured variables are temperature, pH, or pressure, whereas the manipulated variable is the flow rate of a certain material or energy (such as electricity and steam).

Disturbances in the variables are those changes that are not caused by an operator or a control mechanism but result from some change outside the boundaries of the system. Disturbances cause undesirable changes in the output of a system. For example, the temperature of water in a tank is a controlled variable that may be influenced by the inlet flow rate, temperature of the inlet flow, and exit flow rate of water.

Robustness describes how tolerant the system is to changes in process parameters. When the robustness of a control system decreases, a small change in a process parameter makes the system unstable.

Performance communicates the effectiveness of the control system. There is a tradeoff between robustness and performance.

2.11.2 Input and Output Signals to Control Processes

A variety of signals are transmitted between the control system and processing equipment (Figure 2.53). These signals include:

- **Output signals**, which send a command to actuate components of a processing equipment such as valves and motors
- **Input signals** to the controller that
  - **a.** Provide feedback from the processing components when a certain valve or motor has been actuated
  - **b.** Measure selected process variables including temperature, flow, and pressure
  - **c.** Monitor the processing equipment and detect completion of a certain process

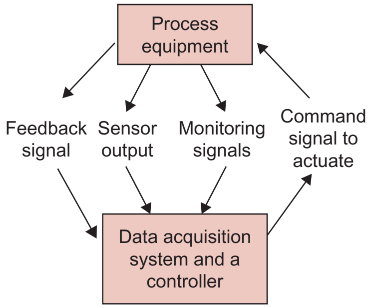


Figure 2.53 Communication between process equipment and data acquisition system/controller.

Signals received by a controller are analyzed by following certain logic that is programmed in the control system, similar to the logic a human operator may follow in controlling a process. The overall aims of controlling a processing system are to minimize the effect of any external disturbances, operate the process under stable conditions, and achieve optimal performance. We will examine different strategies used in design of a control system in the following section.

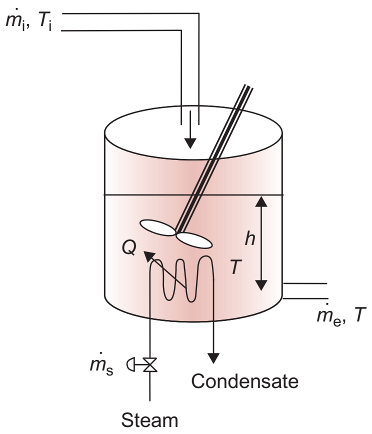
2.11.3 Design of a Control System

2.11.3.1 *Control Strategy*

A control system may be designed to provide digital or analog control, or monitor tasks. For example, processing equipment may be set up under digital control so that it can be turned on or off from a control panel located at a remote location. Similarly, valves may be opened or closed, or the operation of multiple pieces of equipment may be carried out in some desired sequence.

An analog control is obtained via analog signals sent from the control unit. When combined with a feedback signal, analog controls are useful to operate valves that may be partially closed or opened. For example, the flow of steam or hot or cold water to processing equipment may be controlled.

Monitoring allows for checking critical aspects of the process for any major faults. Upon receiving a signal indicating a fault, the equipment can be shut down or the process stopped until the fault is corrected. Data acquisition is another feature of the automation



■ **Figure 2.54** A tank of water heated with an indirect steam heat exchanger.

system where the collected data can be used by the plant management to improve process efficiencies, or conduct scheduled maintenance, quality assurance, and cost analysis.

2.11.3.2 *Feed Backward Control System*

Consider a simple case of heating water in a tank (Figure 2.54). The tank is equipped with a steam coil and an agitator. The purpose of the agitator is to provide good mixing so that the temperature of water is uniform inside the tank—that is, it does not vary from one location to another inside the tank. Steam is conveyed to the coil by first passing it through a control valve. Steam condenses inside the coil and the heat of condensation, Q, is discharged into the water surrounding the coil while the condensate exits the coil. Water at a rate of $\dot{m}_i$ (kg/s) at a temperature of T_i is pumped into the tank and exits the tank at a rate of $\dot{m}_e$ (kg/s) with an exit temperature T (same as the temperature of water in the tank). The height of water in the tank is h. When operating this water heater, it is important to ensure that the volume of the water in the tank is maintained at some predetermined level; it should not overflow or run empty. Likewise, the temperature of the water exiting the tank must be maintained at some desired value.

Under steady state conditions, this water heating system should operate well if there are no changes in the inlet flow rate of water ($\dot{m}_i$) or its temperature (T_i). What if there is a change in either $\dot{m}_i$ or T? This will cause a disturbance in the process and will require intervention. If the process is supervised by an operator who is checking the temperature and notices a change (disturbance), the operator will try to change the steam flow rate by closing or opening the steam valve. This simple description of the process implies that we cannot leave this system to operate on its own. It requires some type of either manual supervision or an automatic control.

The objective of a control system is to determine and continuously update the valve position as the load condition changes. In a feedback control loop, the value of a controlled variable is measured and compared against a desired value usually called the set value. The difference between the **desired** and **set** value is called **controller error**. The output from the controller, which is a function of the controller error, is used to adjust the manipulated variable.

Next, let us consider a temperature control that may substitute for manual supervision.

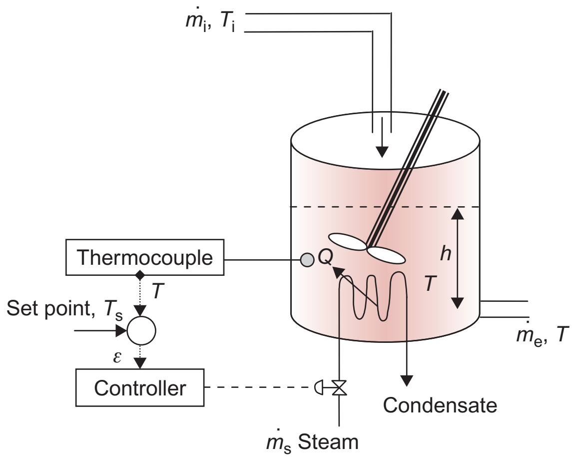


Figure 2.55 A feedback control system used in a tank of water heated with an indirect steam heat exchanger.

As seen in Figure 2.55, a temperature sensor (thermocouple) is installed in the tank. The steam valve and the thermocouple are connected by a controller. The objective of this controller is to keep the temperature of the water constant at T_s (a set point temperature) whenever there is a change in the flow rate $\dot{m}_i$ or the inlet temperature T_i. In this arrangement, when the thermocouple senses a change in temperature, say ε, where

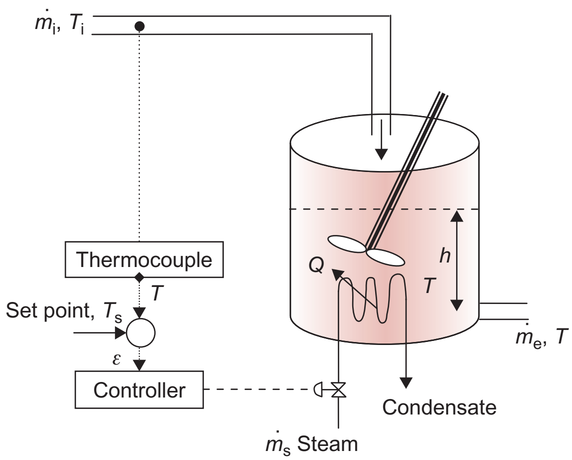
$$\varepsilon = T_s - T \tag{2.201}$$

the deviation ε is conveyed to the controller. If the deviation ε is greater than zero, meaning that the temperature of the water has fallen and should be increased, the controller sends a signal to the steam valve, opening it to allow steam to be conveyed to the coil. When the temperature of water reaches the desired value T_s, the controller shuts off the steam valve.

The control shown in Figure 2.55 is called a feedback control because the signal conveyed by the controller to the steam valve occurs after measuring the water temperature and comparing it with the set point temperature.

2.11.3.3 *Feedforward Control System*

Another type of control is a feedforward control, as shown in Figure 2.56. In this case the thermocouple is installed in the inlet feed pipe. When the temperature of the feed water (T_i) decreases below a set point (T_s), it implies that deviation ε will be greater than $T_s - T_i$, and it will cause the water temperature in the tank to decrease.

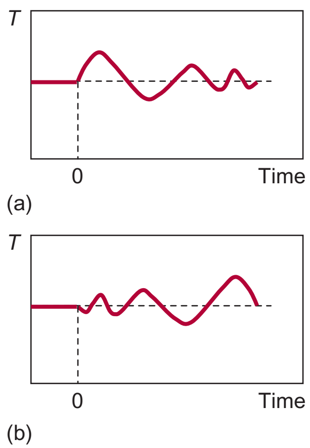


■ **Figure 2.56** A feedback control system used in a tank of water heated with an indirect steam heat exchanger.

In this case, the controller sends a signal to the steam valve, causing the valve to open and resulting in more steam flowing to the coil.

The difference between the feedforward and feedback controls should be clear by observing that in a feedforward control the controller anticipates that there will be a change of water temperature in the tank because of the change in the inlet water temperature, and a corrective action is taken prior to any observed change of water temperature in the tank.

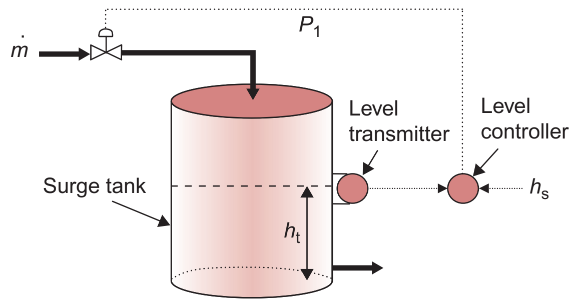
Similar controls may be installed to monitor the height of water (h) in the tank and maintain it at a set point by controlling the feed rate of water into the tank.



■ **Figure 2.57** Response of a system: (a) stable, (b) unstable.

2.11.3.4 *Stability and Modes of Control Functions*

Another important reason for using automatic controls is to maintain the stability of a system or process. In a stable system, a disturbance (i.e., change) in any variable should decrease with time, as shown in Figure 2.57a. For example, if the temperature goes above a certain set point and a system is able to bring it back to the initial value, an external means of correction is not required. In an unstable system (Fig. 2.57b), a disturbance in a variable would continue to increase to the point where the system is unable to bring the variable back to its original value on its own, requiring some external corrective action. Thus a controller is required to prevent instability in a system.



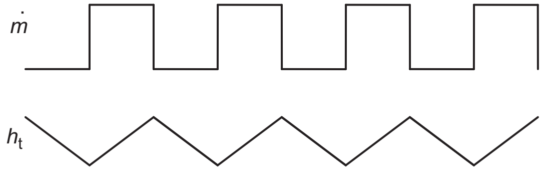
■ **Figure 2.58** Control of water level in a surge tank.

To understand the operating principles of different types of controllers, let us consider controlling the height of liquid in an intermediate storage tank. Intermediate storage tanks are often used to act as a buffer between two different processes. As shown in Figure 2.58, water is pumped through a control valve into the tank, and it exits at the bottom. To ensure that the tank does not overflow, a level sensor is installed to measure the height of the water in the tank, h_t. The sensor is connected to a level controller. The desired height in the tank, called the set point, is h_s. The difference between the desired height and measured height is the error. If the error is zero, then no control action is required. However, if an error is present, then the level controller sends a signal to a valve installed at the inlet pipe. A common signal to operate the valve is pneumatic pressure, P_1, required to operate the valve to open it. With this set up, we will now consider different types of controllers that can be used.

2.11.3.5 *On-Off Control*

An on-off control is similar to that used on a household thermostat for an air-conditioning system. The control is either maximum or zero flow. In our application, shown in Figure 2.58, the on-off controller sends a signal to the control valve when the error indicates that the level in the tank has decreased. The signal results in the application of pneumatic pressure P_1 on the control valve, and the valve opens. This is called a *fail-closed* valve since it remains closed until pressure is applied to open it. Such a valve will prevent water from flowing into the tank if the control system breaks down.

Many of the control valves used in the industry operate with signals between 3 and 15 psig. This means that a *fail-close* valve will be fully closed at 3 psig and fully open when a pressure of 15 psig is applied.



■ **Figure 2.59** A cyclic response to on-off control signal.

An on-off controller is generally avoided for a continuous operation because it yields a cycling response (shown in Fig. 2.59), which is not desirable. There is also more wear on the control valves in this system, due to excessive operation.

2.11.3.6 *Proportional Controller*

In a proportional controller, the actuating output of a controller (c) is proportional to the input error (ε), the difference between the set point and the measured values. Thus,

$$c(t) = G_p \varepsilon(t) + c_s \tag{2.202}$$

where G_p is the proportional gain of the controller and c_s is the controller's bias signal. The bias signal is the actuating signal when error is zero. Thus, in our example the bias signal is the steady state pressure applied on the valve when the error is zero.

The proportional gain (G_p), may be viewed as an amplification of the signal. In our example, a positive error, or when the set point is greater than the measured height, results in an increase in the flow rate into the tank. The flow rate will increase only if the pressure applied on the valve is increased. For a given amount of error, as the proportional gain is increased, it will result in more of the control action.

In some controllers, a proportional band is used instead of proportional gain. The proportional band represents the range of error that causes the output of the controller to change over its entire range. The proportional band is given by the following equation:

$$\text{Proportional Band} = \frac{100}{G_p} \tag{2.203}$$

In a proportional controller, an offset between the set point and the actual output is created when the controller output and the process output arrive at a new equilibium value even before the error diminishes to zero. To prevent the offset, a proportional integral controller is used.

2.11.3.7 Proportional Integral (PI) Controller

In a proportional integral (PI) controller, both error and the integral of the error are included in determining the actuating signal. Thus, for a PI controller, the actuating signal is

$$c(t) = G_p\varepsilon(t) + G_i\int_0^t \varepsilon(t)dt + c_s \quad (2.204)$$

The first term in the right-hand side of the equation is proportional to the error and the second term is proportional to the integral of the error. In a PI controller, an error signal will cause the controller output to change in a continuous manner, and the integral of the error will reduce the error to zero. In the integral action, the output of a controller is related to the time integral of the controller error, making the controller output dependent upon the size and duration of the error. Thus, with the integral function the past history of the controller response is accounted for in determining the actuating signal. For this reason, PI controllers are preferred in many applications.

2.11.3.8 Proportional–Integral–Derivative Controller

In a proportional–integral–derivative (PID) controller, the current rate of change or the derivative of the error is incorporated. The output of a PID controller is as follows:

$$c(t) = G_p\varepsilon(t) + G_i\int_0^t \varepsilon(t)dt + G_d\frac{d\varepsilon(t)}{dt} + c_s \quad (2.205)$$

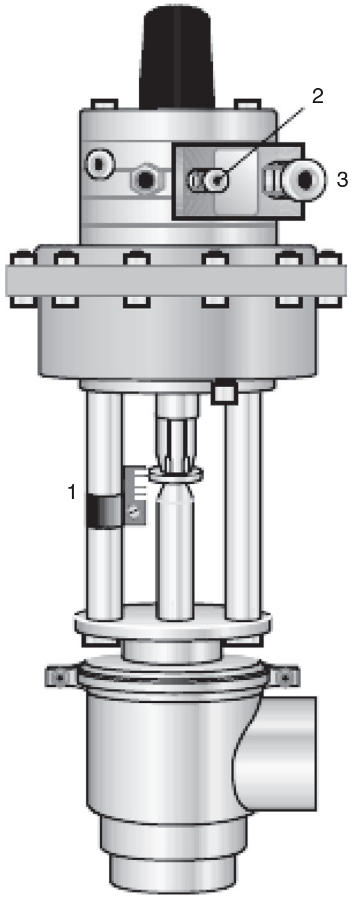
A PID controller can be viewed as an anticipatory controller. The current rate of change of error is used to predict whether the future error is going to increase or decrease, and this information is then used to determine the actuating output. If there is a constant nonzero error, then the derivative control action is zero. PID controllers are not suitable when there is noise in the input signal, since a small nonzero error can result in an unnecessary large control action.

2.11.3.9 Transmission Lines

Transmission lines are used to carry the signal from the measurement sensor to the controller and from the controller to the control element. Transmission lines are electric or pneumatic using compressed air or liquid.

2.11.3.10 Final Control Element

The final control element carries out the implementing action. Upon receiving a signal, the control element will adjust. The most common



1. Visual position indicator
2. Connection for electrical signal
3. Connection for compressed air

Figure 2.60 A pneumatic valve. (Courtesy Alpha Laval)

control element used in the food industry is a pneumatic valve, as shown in Figure 2.60. This valve is operated with air and controls the flow by positioning a plug in an orifice. The plug is connected to a stem that is attached to a diaphragm. With a change in the air pressure due to the control signal sent by the controller, the stem moves and the plug restricts the flow through the orifice. This is the principle behind an air-to-close valve. If, due to some failure, the air supply is lost, the valve will fail open because the spring would push the stem and the plug upward. In addition, there are air-to-open with fail-closed valves. These valves will be fully open or closed as the air pressure on the side of the diaphragm changes from 3 to 15 psig (20 to 100 kPa).

An accurate measurement of controlled output is necessary for any feedback control system. The most common controlled variables in process controls are as follows: temperature, pressure, flow rate, composition, and liquid level. A large number of commercial sensors, based on numerous operating principles, are available to measure these variables. Some of the variables are measured directly, such as pressure. Temperature is measured indirectly; for example, in a thermocouple, temperature change is converted into voltage. The output signal of a sensor is often converted into another signal for transmission. For signal transmission the common standard systems are shown in Table 2.7.

The output of sensors may be changed to the same type of signal; for example, both temperature and pressure signals may be changed to a current of 4–20 mA carried on different pairs of wires. When a signal arrives at the processing unit, it may be necessary to amplify it to operate a solenoid valve or to change the position of a control valve.

During transmission, electrical signals may be corrupted by extraneous causes, such as by the operation of nearby large electrical equipment. External noise can be eliminated using appropriate

Table 2.7 Standard Systems Used for Signal Transmission

Pneumatic	Pneumatic (3–15 psig or 20–100 kPa)
Electric Current (direct current)	4–20 mA
Voltage (d.c.)	0–10 V
	0–5 V

filters. Low signal strength (e.g., mV signal) is more prone to extraneous noise than higher signal strength (V signal). Voltage signals are less robust than electric current signals, and digital signals are less prone to external electrical noise.

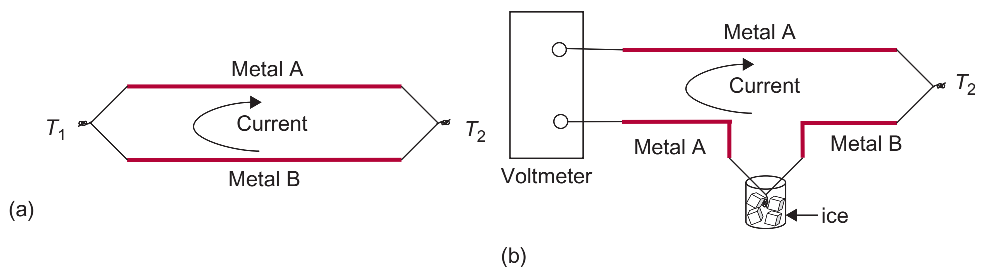
2.12 SENSORS

As we have learned in the preceding discussion, sensors play an important role in measuring process variables. Next, we will review the operating principles of some common sensors used in food processing.

2.12.1 Temperature

Temperature sensors used in the industry may be classified broadly as indicating and recording type. The common indicating sensors used in measuring temperature are a bimetallic strip thermometer and filled thermal systems. For recording and process control purposes, the most common types of temperature sensors are thermocouples, thermistors, and resistance temperature detectors. These three sensors provide an electrical signal based on the measurement.

A thermocouple is a simple device containing an electrical circuit made with two wires of dissimilar materials. The wires are joined at the ends to create two junctions, as shown in Figure 2.61a. If the two junctions are held at different temperatures, a difference in electric potential is created, resulting in a flow of current in the circuit (called the Seebeck effect). The potential difference (typically on the order of mV) measured in this circuit is a function of the temperature



■ **Figure 2.61** Thermocouple circuit: (a) Current is generated when temperature $T1 \neq T2$; (b) When one junction is placed in ice then the temperature of second junction can be measured.

Table 2.8 Some Common Combinations of Metals and Alloys Used in Thermocouples

Thermocouple type	Composition of positive wire	Composition of negative wire	Suitability (Environment, limits of temperature)
J	Iron	Constantan	Oxidizing, reducing, inert, up to 700°C
K	90% Ni 10% Cr	95% Ni 5% Al	Oxidizing, inert, up to 1260°C
T	Copper	Constantan	Oxidizing, vacuum, reducing or inert, up to 370°C, suitable for −175°C.

difference between the two junctions. In the circuit shown in Figure 2.61a, only temperature difference can be determined. To determine the unknown temperature we need to know the temperature of one of the junctions. This can be accomplished by keeping one of the junctions in ice, as shown in Figure 2.61b. Alternatively, an electronic ice junction is built into the circuit to determine the absolute temperature. Table 2.8 lists some common combinations of metals and alloys used in thermocouples.

A resistance temperature detector (RTD) provides high accuracy and is useful when a difference in temperatures is to be measured. Either a thin film or wire resistor is used. Standard resistance of 100, 500, or 1000 ohms is most common. Typically, RTDs are made of either platinum or nickel. Thus, a Pt100 refers to an RTD made of platinum with 100 ohm resistance. Since an RTD is a very thin film or wire, it is embedded in another material to make it suitable for mechanical handling. Typically ceramic is used for this purpose, as shown in Figure 2.62. Performance features of thermocouples and RTDs are presented in Table 2.9.

In process equipment such as tanks and pipes, temperature is measured by inserting the sensor in a thermowell, as shown in Figure 2.63. Thermowells isolate the temperature sensor from the surrounding environment. They are designed to prevent performance degradation of the temperature sensor. The length of the thermowell should be at least

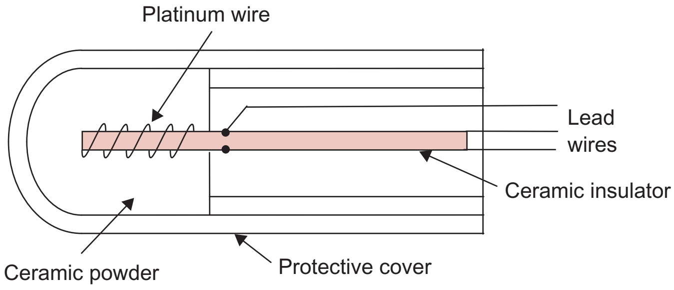


Figure 2.62 A resistance temperature device.

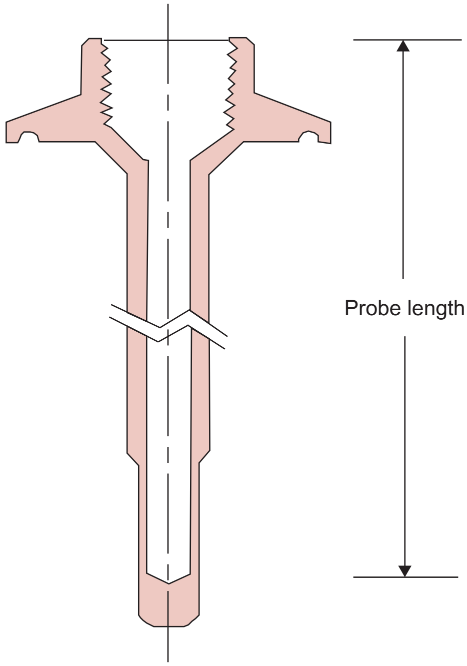
Table 2.9 Performance of Thermocouples and Resistance Temperature Detectors

Performance	Thermocouple	Resistance temperature detector (RTD)
Operating temperature range	Wide	Wide, −200 to +850°C
Time response	Fast (faster than RTD)	Slow
Functional relation with temperature	Nonlinear	Linear, well-defined relationship
Durability	Simple and rugged	Sensitive to vibration, shock, and mechanical handling
Sensitivity to electromagnetic interference	Sensitive to surrounding electromagnetic interference	
Reference	Reference junction is required	No
Cost	Inexpensive	Expensive

15 times the diameter of the thermowell tip. The dynamic response of a temperature sensor when located in a thermowell can be quite long and therefore problematic in undermining the control performance.

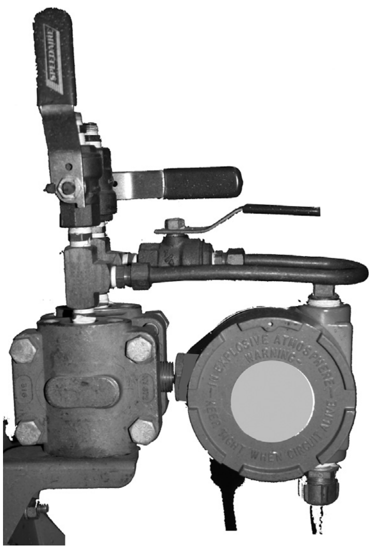
2.12.2 Liquid Level in a Tank

The level of liquid in a tank may be measured using different methods. One method is to use a float that is lighter than the fluid in the tank,



■ **Figure 2.63** A thermowell used to isolate a temperature sensor from the process environment.

similar to the valve used in a bathroom toilet. In another system, the apparent weight of a cylinder is measured as it is buoyed up or down in a tank of liquid. Yet another method involves measurement of the differential pressure between two locations, one in the vapor and the other in the liquid. There are also ultrasonic level detectors that are used to measure liquid level in a tank.



■ **Figure 2.64** A differential pressure transmitter.

2.12.3 Pressure Sensors

Pressure sensors are used to measure pressure in the equipment and also to measure pressure differences to determine the height of a liquid in a tank or flow rates of liquids and gases. In a variable capacitance differential pressure transducer, the pressure difference causes displacement of the diaphragm, which is sensed by capacitor plates located on both sides of the diaphragm. The differential capacitance between the sensing diaphragm and the surrounding capacitor plates is converted into DC voltage. For indicating purposes, manometers and a Bourdon gauge are commonly used. However, for recording pressures, a pressure sensor capable of generating an electrical signal is preferred. One type of pressure sensor with electrical signal output is shown in Figure 2.64. Inside this instrument, a flexible diaphragm is the pressure transmitting element. The pressure acting on the diaphragm causes a rod to push a flexible beam. Two strain gages are mounted on the beam. The deflection of the beam is sensed by the strain gages, and the reading is converted to a pressure reading.

2.12.4 Flow Sensors

Liquid and gas flow are commonly measured by passing the fluid through a constriction that causes a pressure drop. The pressure drop is measured and the flow rate is determined using a Bernoulli equation as previously described in this Chapter. Typical constrictions used in flow measurement are an orifice plate and venturi tube. Another type of flow sensor is a turbine flow meter, where the number of turbine revolutions is determined to calculate the flow rate. For air flow, a vane-type anemometer is used to count the number of revolutions. For measuring air flow, hot-wire anemometers are commonly used.

In flow meters installed in-line, the material of the meter comes into direct contact with the food medium. Therefore, hygienic design and construction (that allow ease of cleaning) and minimization of any dead spaces are vital in food applications. If the flow measuring device can be cleaned in situ, then it must withstand the conditions employed during clean-in-place procedures. If the device cannot be cleaned in situ, then it must be easy to dismantle for cleaning purposes.

Typical flow meters used in the food industry include turbine, positive displacement, electromagnetic, and sensors based on the Coriolis principle.

In positive displacement meters, fixed volume chambers rotate around an axis. The liquid fills in the inlet chamber and causes the rotation. The rotation of these meters is linked to some type of mechanical counter or electromagnetic signal. The advantage of these meters is that they do not require an auxiliary source of energy as the moving fluid provides the movement of the sensor, and they can be installed without requiring any special length of upstream straight section. They operate well with a range of viscosities and are good at low flow rates. Typical fluids using positive displacement meters are edible oils and sugar syrups. Turbine meters are more suitable for low viscosity fluids such as milk, beer, and water. Their rotary motion is obtained by conversion of the free stream energy. A turbine is installed so that it forms a helix, and every revolution of helix is equivalent to the length of the screw. Turbine meters should be installed away from upstream fittings.

2.12.5 Glossary of Terms Important in Data Acquisition

Many of the following terms are encountered in experimental measurements and data analysis. Brief definitions are provided so that these terms are correctly used in reporting results from experimental measurements.

Accuracy: The difference between an indicated and an actual value.
Drift: A change in the reading of an instrument of a fixed variable over time.
Error Signal: The difference between the set point and the amplitude of measured value.
Hysteresis: The difference in readings obtained when an instrument approaches a signal from opposite directions.
Offset: A reading of instrument with zero input.
Precision: The limit within which a signal can be read.
Range: The lowest and highest value an instrument is designed to operate or that it can measure.
Repeatability: A measure of closeness of agreement between a number of readings taken consecutively of a variable.
Resolution: The smallest change in a variable to which the instrument will respond.
Sensitivity: A measure of change of output of an instrument for a change in the measured variable.

2.13 DYNAMIC RESPONSE CHARACTERISTICS OF SENSORS

As we learned in the previous sections, a variety of sensors are used in control and measurement of variables in food processing. For example, a thermocouple, thermometer, or a thermistor is used for measuring temperature. Similarly, other properties such as pressure, velocity, and density are frequently measured with appropriate sensors. In selecting a sensor, a key criterion is its dynamic response. We will briefly review the dynamic response characteristics of sensors.

The dynamic response characteristic of any sensor is measured by determining its time constant, which provides us with a measure of how fast or slow a given sensor responds to a change in input. For example, if we are measuring the temperature of a liquid food as it is being heated, we need to know how much time lag exists between the actual temperature of the liquid and the temperature indicated by a sensor placed in the liquid food. If the liquid temperature is changing rather slowly, then this time lag may not be a major problem. But, if the liquid is heating rapidly, we should select a sensor that can respond without much lag time. Quantitatively, we describe this lag period by determining the *time constant* of the sensor.

The aforementioned terms "rapid" and "slow" in instrument response are subjective and are not very useful in selecting sensors. What we need is an objective method to determine whether a selected sensor will be suitable for a given job. For this purpose, we determine the time constant of a temperature sensor. Furthermore, the same methodology may be used for other types of sensors, such as for measuring pressure, velocity, and mass.

The time constant is expressed in units of time, such as seconds. The time constant is a function of a given sensor.

To determine the time constant, note that the response of a given sensor to a sudden change in input is exponential. As shown in Figure 2.65, the temperature response of a thermometer follows an exponential plot when the surrounding temperature is suddenly changed from 0°C to 25°C. The time constant, τ, is calculated using the following equation that describes such an exponential plot:

$$T = T_u - (T_u - T_0)e^{-\frac{t}{\tau}} \tag{2.206}$$

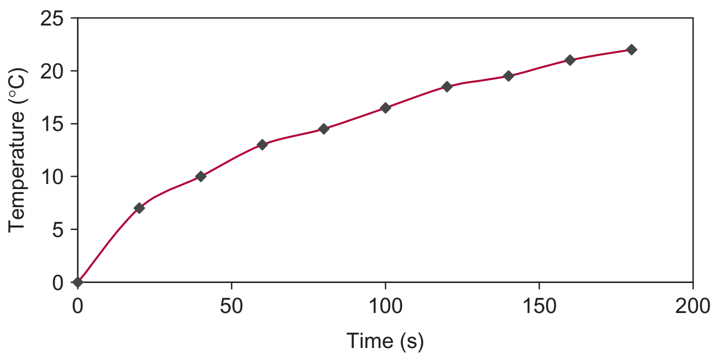


Figure 2.65 Exponential response of a temperature sensor.

where T is the sensor temperature (°C), T_u is the ambient temperature (°C), T_0 is the initial temperature (°C), t is time (*s*), and τ is time constant (*s*).

Rearranging Equation (2.206)

$$\frac{T_u - T}{T_u - T_0} = e^{-\frac{t}{\tau}} \tag{2.207}$$

Taking natural logarithm of both sides, we get

$$\ln \frac{T_u - T}{T_u - T_0} = -\frac{t}{\tau} \tag{2.208}$$

Equation (2.208) is that of a straight line, $y = mx + c$ where y-ordinate is

$$\ln \frac{T_u - T}{T_u - T_0}$$

and x-axis is t, and the time constant, τ, is obtained from the inverse of the negative slope.

Subtracting both left- and right-hand sides of Equation (2.207) from 1, we obtain

$$1 - \frac{T_u - T}{T_u - T_0} = 1 - e^{-\frac{t}{\tau}} \tag{2.209}$$

Rearranging,

$$\frac{T - T_0}{T_u - T_0} = 1 - e^{-\frac{t}{\tau}} \tag{2.210}$$

Therefore, at time equal to one time constant, that is, $t = \tau$,

$$\frac{T - T_0}{T_u - T_0} = 1 - e^{-\frac{\tau}{\tau}} = 1 - e^{-1} = 1 - 0.3679 = 0.6321$$

or

$$T = T_0 + 0.6321(T_u - T_0)$$

This implies that at a time equal to one time constant, the temperature of the sensor would have increased by 63.21% of the step temperature change, $T_u - T_0$. Example 3.10 illustrates how the time constant is obtained from experimental data.

Example 2.28

An experiment was conducted to measure the time constant of a bimetallic sensor. The sensor was initially equilibrated in ice water. The experiment began by removing the sensor from ice water. Any residual water from the sensor was quickly wiped and it was held in a room with an ambient temperature of 23°C. The following data on temperature versus time were obtained at 20-s intervals. Using these data, estimate the time constant.

Time (s)	Temperature (°C)
0	0
20	7
40	10
60	13
80	14.5
100	16.5
120	18.5
140	19.5
160	21
180	22

Given

Initial temperature, $T_0 = 0°C$
Ambient temperature, $T_u = 23°C$

Approach

We will use a spreadsheet to solve this problem. Equation (2.208) will be programmed and the slope of a straight line will give the time constant value.

Solution

1. *Program a spreadsheet as shown in Figure E2.11.*
2. *Create a plot between* $\ln \frac{(T_u - T)}{(T_u - T_0)}$ *and time.*
3. *Use trend line feature to calculate slope. As shown in the figure, the slope is*

$$Slope = -0.0149$$

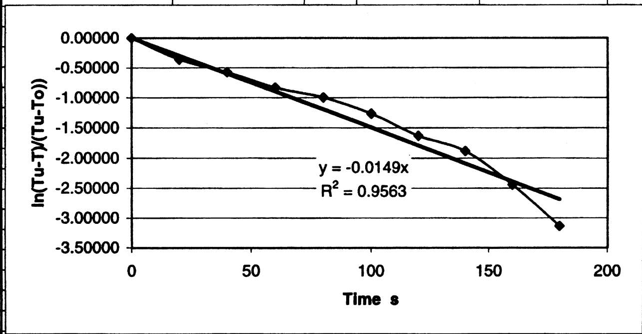
$$Timeconstant = -\left(\frac{1}{slope}\right)$$

$$= 67.1\ s$$

4. *The time constant of the given bimetallic sensor is 67.11 s.*

Figure E2.11 Spreadsheet for calculating time constant for data given in Example 2.28.

	A	B	C	D	E	F
1	Initial Temperature, To	23				
2						
3	Time(s)	Temperature (C)	(Tu-T)	(Tu-T)/(Tu-To)	ln((Tu-T)/Tu-To))	
4						
5	0	0	23	1.0000	0.00000	
6	20	7	16	0.6957	-0.36291	
7	40	10	13	0.5652	-0.57054	
8	60	13	10	0.4348	-0.83291	
9	80	14.5	8.5	0.3696	-0.99543	
10	100	16.5	6.5	0.2826	-1.26369	
11	120	18.5	4.5	0.1957	-1.63142	
12	140	19.5	3.5	0.1522	-1.88273	
13	160	21	2	0.0870	-2.44235	
14	180	22	1	0.0435	-3.13549	
15						



PROBLEMS

2.1 Calculate the Reynolds number for 25°C water flow in a 1-in nominal diameter sanitary pipe at 0.5 kg/s. What are the flow characteristics?

2.2 A pipe discharges wine into a 1.5-m-diameter tank. Another pipe (15 cm diameter), located near the base of the tank, is used to discharge wine out of the tank. Calculate the volumetric flow rate into the tank if the wine level remains constant at 2.5 m.

2.3 Apple juice at 20°C is siphoned from a large tank using a constant-diameter hose. The end of the siphon is 1 m below the bottom of the tank. Calculate the height of the hill over which the hose may be siphoned without cavitation. Assume properties of apple juice are same as water. The atmospheric pressure is 101.3 kPa. Height of the juice in the tank is 2 m.

2.4 Sulfuric acid with a density of 1980 kg/m^3 and a viscosity of 26.7 cP is flowing in a 35-mm-diameter pipe. If the acid flow rate is 1 m^3/min, what is the pressure loss due to friction for a 30-m length of smooth pipe?

2.5 Compute the mean and maximum velocities for a liquid with a flow rate of 20 L/min in a 1.5-in nominal diameter sanitary pipeline. The liquid has a density of 1030 kg/m^3 and viscosity of 50 cP. Is the flow laminar or turbulent?

2.6 Calculate the total equivalent length of 1-in wrought iron pipe that would produce a pressure drop of 70 kPa due to fluid friction, for a fluid flowing at a rate of 0.05 kg/s, a viscosity of 2 cP, and density of 1000 kg/m^3.

***2.7** A solution of ethanol is pumped to a vessel 25 m above a reference level through a 25-mm-inside-diameter steel pipe at a rate of 10 m^3/h. The length of pipe is 30 m and contains two elbows with friction equivalent to 20 diameters each. Compute the power requirements of the pump. Solution properties include density of 975 kg/m^3 and viscosity of 4×10^{-4} Pa s.

2.8 The flow of a liquid in a 2-in nominal diameter steel pipe produces a pressure drop due to friction of 78.86 kPa. The length of pipe is 40 m and the mean velocity is 3 m/s. If the density of the liquid is 1000 kg/m^3, then

a. Determine the Reynolds number.
b. Determine if the flow is laminar or turbulent.
c. Compute viscosity of the liquid.
d. Estimate the temperature, if the liquid is water.
e. Compute the mass flow rate.

***2.9** A pump is being used to transport a liquid food product ($\rho = 1000$ kg/m^3, $\mu = 1.5$ cP) from a holding tank to a filling machine at a mass flow rate of 2 kg/s. The liquid level in the holding tank is 10 m above the pump, and the filling machine is 5 m above the pump. There is 100 m of 2-in nominal diameter sanitary pipeline between the holding tank and the filling machine, with one open globe valve and four regular 90° flanged elbows in the system. The product is being pumped through a heat exchanger with 100 kPa of

* Indicates an advanced level in solving.

pressure drop due to friction before filling. Determine the theoretical power requirement for the pump.

2.10 A centrifugal pump, located above an open water tank, is used to draw water using a suction pipe (8 cm diameter). The pump is to deliver water at a rate of 0.02 m^3/s. The pump manufacturer has specified a $NPSH_R$ of 3 m. The water temperature is 20°C and atmospheric pressure is 101.3 kPa. Calculate the maximum height the pump can be placed above the water level in the tank without cavitation. A food process equipment located between the suction and the pump causes a loss of $C_f = 3$. All other losses may be neglected.

2.11 A centrifugal pump is operating at 1800 rpm against 30 m head with a flow rate of 1500 L/min. If the pump speed is doubled, calculate the new flow rate and developed head.

2.12 Edible oil (specific gravity = 0.83) flows through a venturi meter with a range of flow rates from 0.002 to 0.02 m^3/s. Calculate the range in pressure differences required to measure these flow rates. Pipe diameter is 15 cm and diameter of the venturi throat is 5 cm.

2.13 A pipe (inside diameter 9 cm) is being used to pump a liquid (density = 1100 kg/m^3) with a maximum mass flow rate of 23 kg/s. An orifice plate ($C = 0.61$) is to be installed to measure the flow rate. However, it is desired that the pressure drop due to the orifice plate must not exceed 15 kPa. Determine the diameter of the orifice plate that will ensure that the pressure drop will remain within the stated limit.

2.14 A capillary tube is being used to measure the viscosity of a Newtonian liquid. The tube has a 4-cm diameter and a length of 20 cm. Estimate the viscosity coefficient for the liquid if a pressure of 2.5 kPa is required to maintain a flow rate of 1 kg/s. The liquid density is 998 kg/m^3.

2.15 Calculate the viscosity of a fluid that would allow a pressure drop of 35 kPa over a 5-m length of 0.75-in stainless steel sanitary pipe if the fluid is flowing at 0.12 m^3/h and has a density of 1010 kg/m^3. Assume laminar flow.

2.16 A 2-cm-diameter, 5-cm-long capillary-tube viscometer is being used to measure viscosity of a 10 Pa s liquid food. Determine the pressure required for measurement when a flow rate of 1 kg/min is desired and $\rho = 1000$ kg/m^3.

***2.17** A capillary-tube viscometer is being selected to measure viscosity of a liquid food. The maximum viscosity to be measured will be 230 cP, and the maximum flow rate that can be measured accurately is 0.015 kg/min. If the tube length is 10 cm and a maximum pressure of 25 Pa can be measured, determine the tube diameter to be used. The density of the product is 1000 kg/m^3.

***2.18** A single-cylinder rotational viscometer is used to measure a liquid with viscosity of 100 cP using a spindle with 6 cm length and 1 cm radius. At maximum shear rate (rpm = 60), the measurements approach a full-scale reading of 100. Determine the spindle dimensions that will allow the viscometer to measure viscosities up to 10,000 cP at maximum shear rate.

2.19 A dry food powder with a bulk density of 650 kg/m^3 and an angle of repose of 55° is discharged by gravity through a circular opening at the bottom of a large storage vessel. Estimate the diameter of the opening needed to maintain a mass flow rate of 5 kg/min. The discharge coefficient is 0.6.

2.20 A dry food powder is being stored in a bin with a 10-cm diameter outlet for gravity flow during discharge. The angle of internal friction is 5° and the bulk density is 525 kg/m^3. Estimate the cohesiveness parameter of a powder that would result in bridging when the product is discharged from the storage bin.

***2.21** Zigrang and Sylvester (1982) give several explicit empirical equations for friction factor as a function of the Reynolds number. The equation attributed to Churchill is said to be applicable to all values of N_{Re} and ε/D. Churchill's model is

* Indicates an advanced level in solving.

for the Darcy friction factor, f_D, which is four times the value of the Fanning friction factor, f.

$$f_D = 8\left[\left(\frac{8}{N_{Re}}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12}$$

where

$$A \equiv \left[2.457 \ln\left\{\left(\frac{7}{N_{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}\right\}\right]^{16} \qquad B \equiv \left(\frac{37{,}530}{N_{Re}}\right)^{16}$$

Using MATLAB®, evaluate Churchill's equation for $\varepsilon/D = 0.0004$ and $N_{Re} = 1\times10^4$, 1×10^5, 1×10^6, and 1×10^7. Compare results to readings from the Fanning friction factor chart (Fig. 2.16) and to Haaland's explicit friction factor for turbulent flow given by Equation (2.54).

*2.22 The turbulent portion ($N_{Re} \geq 2100$) of the Moody diagram given in Figure 2.16 is constructed using the Colebrook equation:

$$\frac{1}{\sqrt{f_D}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{N_{Re}\sqrt{f_D}}\right)$$

where f_D is the Darcy friction factor, which is four times the value of the Fanning friction factor, f.

Use the MATLAB® function **fzero** to solve for f_D (and f) in the Colebrook equation for $\varepsilon/D = 0.0004$ and the Reynolds numbers of 2100, 1×10^4, 1×10^5, 1×10^6, and 1×10^7. Compare these results to values read from Figure 2.16.

*2.23 A thin-plate orifice has been installed to measure water flow in a pipe with an inner diameter (D_1) of 0.147 m and an orifice diameter (D_2) of 0.0735 m. The orifice pressure taps are located a distance D_1 upstream and $D_1/2$ downstream of the orifice. Write a MATLAB® script to calculate the flow rate, Q, if the measured pressure drop ($P_A - P_B$) is 9000 Pa when the water temperature is 20°C. Use the procedure outline by White (2008) as shown.

* Indicates an advanced level in solving.

1. Guess a value for $C = 0.61$.
2. Calculate flow $Q(\mathrm{m^3/s})$ using the value of C.

$$Q = \frac{CA_2}{\sqrt{(1-\beta^4)}}\sqrt{\frac{2}{\rho}(p_1 - p_2)}$$

3. Calculate velocity in pipe, u_A, and the Reynolds number, N_{Re}.
4. Calculate a new value for C using the empirical equation for the orifice coefficient for D and $D/2$ taps:

$$C \approx 0.5899 + 0.05\beta^2 - 0.08\beta^6 + (0.0037\beta^{1.25} + 0.011\beta^8)\left(\frac{10^6}{N_{Re}}\right)^{1/2}$$

where

$$N_{Re} = \frac{u_A D_1 \rho_f}{\mu_f} \qquad \beta = \frac{D_2}{D_1}$$

5. Repeat steps (2) through (4) until the solution converges to constant Q.

LIST OF SYMBOLS

A	area ($\mathrm{m^2}$)
α	correction factor for Newtonian fluid
α'	correction factor for non-Newtonian fluid
β	angle of repose
B_A	Arrhenius constant
c	actuating output of a controller
c_s	bias signal
C	coefficient
C_{fe}	coefficient of friction loss during expansion
C_{fc}	coefficient of friction loss due to contraction
C_{ff}	coefficient of friction loss due to fittings in a pipe
C_b	cohesiveness parameter (Pa)
C_g	discharge coefficient in Equation (2.197)
D	pipe or tube diameter (m)
d	particle diameter (micron)
d_c	characteristic dimension (m)
e	volume fraction within a particle
E	internal energy (J/kg)

E_f	frictional loss of energy
$\in$	volume fraction in particle bed
ε	difference between set point and measured value (Equation 2.201)
ε	surface roughness (m)
E	energy (J)
E'	energy per unit mass (J/kg)
E_a	activation energy (J/kg)
E_p	energy supplied by the pump (J/kg)
F	force (N)
f	friction factor
f_c	unconfined yield stress (Pa)
f'_c	flow function
Φ	power (watts)
g	acceleration due to gravity (m/s^2)
G_d	differential gain
G_i	integral gain
G_p	proportional gain
$\dot{\gamma}$	rate of shear (1/s)
H	height (m)
$H(\theta)$	exit geometry function
h	head (m)
j	exponent in Equation (2.43)
K	consistency coefficient (Pa s)
L	length (m)
L_e	entrance length (m)
μ	coefficient of viscosity (Pa s)
$\dot{m}$	mass flow rate (kg/s)
m	mass (kg)
N	rotational speed, revolutions per second
N_p	number of particles
N_{Re}	Reynolds number
N_{GRe}	Generalized Reynolds number
n	flow behavior index
η	efficiency of pump
P	pressure (Pa)
Q	heat added or removed from a system (kJ)
θ	angle
R	radius (m)
ρ	density (kg/m^3)
r	radial coordinate
R_g	gas constant (cal/mol K)

s	distance coordinate along stream line
S	circumference
s_d	standard deviation
σ	shear stress (Pa)
σ_n	normal stress (Pa)
σ'	major principle stress (Pa)
σ_w	shear stress at wall (Pa)
T	temperature (°C)
t	time (s)
τ	time constant (s)
u	fluid velocity (m/s)
$\bar{u}$	mean fluid velocity (m/s)
ν	kinematic viscosity (m^2/s)
V	volume (m^3)
$\dot{V}$	volumetric flow rate (m^3/s)
φ	angle of internal friction
ω	angular velocity (rad/s)
Ψ	porosity
W	work (kJ/kg)
W_m	work done by the pump (kJ/kg)
υ	void
x	distance coordinate in x-direction (m)
y	distance coordinate in y-direction (m)
z	vertical coordinate
Ω	torque (N m)
ΔP	pressure drop (Pa)

Subscripts: a, air ; A, absolute; b, minimum; B, bulk; d, discharge; g, granular; i, inner; m, manometer; n, normal; o, outer; p, particle; s, suction; so, solids; w, wall.

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